YALE ECONOMIC ESSAYS

VOLUME 3 NUMBER 2

The Wealth Effect and Consumption:
A Statistical Inquiry — *John J. Arena*

Income Determinants for the College Graduates
and Return to Educational Investment — *Shane J. Hunt*

Individual Behavior in Oligopolistic Markets:
An Experimental Study — *James W. Friedman*

Individual Choice Under Uncertainty:
An Experimental Study — *F. Trenery Dolbear, Jr.*

On the Application of Mathematics in Soviet Economics
— *Alan Parker*

FALL 1963

YALE ECONOMIC ESSAYS

This journal is intended to serve two principal functions. All doctoral dissertations in economics at Yale University will be published in its pages either in full or in extended summary. It is also a medium for economic comment and analysis by faculty, students, and graduates in economics of Yale University.

Editorial Board: Arthur M. Okun
Henry W. Broude Neil W. Chamberlain Raymond P. Powell

Volume 3 Fall 1963 Number 2

Yale Economic Essays is printed at the Carl Purington Rollins Printing-Office of the Yale University Press. A one-year subscription (two issues) is $4.00. Single copies are $2.50. The rate for students for one year is $2.00. Checks should be made payable to Yale University. Please address any correspondence to the Editorial Board, 701 Sterling–Sheffield–Strathcona Hall, New Haven, Connecticut.

Copyright 1963 by the Yale Economic Essays.

YALE ECONOMIC ESSAYS

TABLE OF CONTENTS

The Wealth Effect and Consumption: A Statistical Inquiry — *John J. Arena* — 251

Income Determinants for the College Graduates and Return to Educational Investment — *Shane J. Hunt* — 305

Individual Behavior in Oligopolistic Markets: An Experimental Study — *James W. Friedman* — 359

Individual Choice Under Uncertainty: An Experimental Study — *F. Trenery Dolbear, Jr.* — 419

On the Application of Mathematics in Soviet Economics — *Alan Parker* — 471

Summary: This paper first examines the definition of and theoretical implications of the Pigou effect and then sets forth a theoretical model of consumption whose basic hypothesis is that net worth and changes in net worth due to price-level movements (capital gains) should be positively related to consumption. This hypothesis is then tested empirically by using regression analysis upon time-series observations from 1901 through 1958.

The statistical results clearly confirm the hypothesis that higher net worth encourages higher levels of consumption. The results are not as conclusive for the capital gains component, but they do imply that such gains positively affect consumption. Income is another important variable whose effect upon consumption was strongly confirmed by the tests.

Secondary hypotheses concerning additional explanatory variables were also examined and tended to be supported by the data. Such secondary hypotheses are as follows: (1) Holdings of liquid assets in the long run do not affect consumption but may be related to it in shorter periods. (2) The more unequal the distribution of income (concentration at higher income brackets), the lower will be aggregate consumption. (3) The higher the per cent of population of retirement age, the greater will be aggregate consumption.

This essay was submitted to Yale University in 1963 in partial fulfillment of the requirements for the Degree of Doctor of Philosophy. The author is currently on the research staff of the Federal Reserve Bank of Boston.

Dissertation Committee: Professors William Fellner (Chairman), Harold Watts and James Tobin.

*

I would like to express my sincere appreciation to Professors James Tobin and Raymond Goldsmith for their invaluable help in this study. It was primarily from Professor Tobin's writings and advice that I conceived of this project, and it was from Professor Goldsmith that I obtained the necessary data to complete the study. I would also like to thank the Ford Foundation and the Cowles Foundation for Research in Economics for their financial assistance in meeting expenses of the research.

My greatest obligation is owed to Professor William Fellner, Professor Harold Watts, and Professor James Tobin, for their encouragement, advice, and patience. To them any merits of this study must be credited; any deficiencies are entirely my own responsibility.

J.J.A.

THE WEALTH EFFECT AND CONSUMPTION: A STATISTICAL INQUIRY [1]

John J. Arena

SECTION I

Capital Gains, Net Worth, and Consumer Choice

The concept of the so-called Pigou or wealth effect has focused upon the change in real value of net monetary assets due to price movements and the effect of this wealth change upon consumption. It is the intention of the present section to define and to extend this wealth effect to all assets in the portfolio of the consumer and to place this complete wealth effect in the framework of a general consumption model similar to that of Modigliani and Brumberg.[2]

This section will first discuss the wealth effect and define it in a simple static framework. It will then consider some of the implications of this effect upon consumption in the static model. The static model will then be extended to one of longer time horizon. And finally from the long-period model a one-period consumption function will be derived which includes this Pigou-effect variable.

A. Wealth Effect and Capital Gains

When Pigou and Patinkin talked of a wealth effect which would restore full employment, given full flexibility of prices,[3] they focused upon the net monetary portion of asset holdings,[4] and claimed that a falling price level, by increasing the purchasing power of these assets, would increase consumption, and that prices would continue to fall until aggregate demand was at the full employment level. The emphasis was upon the aggregate private sector which

1. Since the completion of this study, a somewhat similar work was published by Professors Ando and Modigliani, "The 'Life Cycle' Hypothesis of Saving," in the March 1963 issue of The American Economic Review.

2. Franco Modigliani and Richard Brumberg, "Utility Analysis and the Consumption Function: An Interpretation of Cross-Section Data," in Post-Keynesian Economics, K. Kurihara, ed. (London: George Allen and Unwin, Ltd., 1955), pp. 388–436. Franco Modigliani and Richard Brumberg, "Utility Analysis and Aggregate Consumption Functions: An Attempt at Integration," an unpublished paper.

3. A.C. Pigou, "The Classical Stationary State," Economic Journal (December 1943), pp. 343–51, and "Economic Progress in a Stable Environment," Economica (August 1947), pp. 180–88. Don Patinkin, "Price Flexibility and Full Employment," American Economic Review (September 1948), pp. 534–64.

4. They used net monetary assets which equalled the value of monetary assets (defined in this study as assets of fixed money value) less the value of monetary liabilities. Since most liabilities of the consumer are in fixed money terms, these net monetary assets were a portion of net worth, the other portion consisting of the assets not considered, namely, capital assets.

was assumed to be a net creditor, i.e., to be the possessor of net positive monetary assets. However, the concept of the wealth effect would apply as well to the individual consuming unit.

The basic static model has a small, finite time period. At the beginning of this period the consumer possesses a stock of wealth or net worth which consists of his net monetary assets (monetary assets less liabilities) and his capital assets. Each asset is treated as if it were a composite homogeneous good, M and K, with its own price level, P_M and P_K.

In addition to these two commodities, there are two flows throughout the period that the consumer must consider. The first is time itself or leisure, L. This time can be traded off for income and has a price, P_L, which is the wage rate. This flow cannot be accumulated into a stock for the next time period. The second flow is consumption, C, which is treated as a normal composite good with its own price level, P_C. If the work week (the number of hours of leisure traded off at P_L) is set by tradition or contract, then clearly the consumer's income for the period is determined in direct proportion to the wage rate.

Within this framework the Pigou effect is easily defined. Since the price of money, P_M, is the unit of account (set at 1), it remains the fixed numeraire. However, as P_C falls, capital gains are received by the consumer on his initial stock of net monetary assets. The purchasing power of this stock at the consumer's disposal increases; he will wish to undertake more consumption in the period.

Patinkin makes explicit allowance for this effect in the following consumption function: $C = g(Y, r, \frac{Mo}{P})$. Real consumption is a function of real income, of the interest rate (viewed as the standard time-preference influence on the margin of choice between saving and consumption), and of the real value of initial monetary holdings. The real value of these money holdings varies inversely with changes in P. The marginal propensity to consume out of such real balances is assumed to be positive in the aggregate, but less than one. If P falls, then the initial balances are increased. (P here is P_C).

However, this is not the only source of a wealth effect. If P_K alone increases (or P_C falls relative to P_K) an analogous situation will occur. The consumer again has more real resources. He is wealthier; his stock of wealth is larger; therefore, he will tend to undertake more present consumption.[1] This phenomenon is a wealth effect; it is the same as the Pigou effect only carried over to the capital stock. A comprehensive definition of the Pigou effect should include both types of wealth changes due to price movements. In fact Patinkin makes reference to the Pigou effect on capital assets, but ignores it due to the assumptions of his model. He states, "this necessity to supplement the already familiar real-balance and real-indebtedness effects with an analogous 'physical-asset' effect is obviated by restricting our analysis of a production economy to a period of time during which net investment

1. One might assert that the greater the stock of wealth, ceteris paribus, in the possession of an individual, the smaller is the value placed on each additional increment (saving) of wealth (diminishing marginal utility.) Therefore, saving would fall, i.e., consumption would rise.

is small relative to the existing stock of physical assets."[1]

He goes on to state, "This also requires us to assume that the prices of these assets always change in the same proportion as the general price level."[2] (P_C and P_K change in proportion). "...The only commodities that appear in this market are finished ones. These are divided into two categories: consumer commodities and investment commodities. The prices of these two categories are assumed to change in the same proportion."[3] In other words, there can be no real capital gains, or wealth effect, on capital goods.[4]

However, this assumption of Patinkin to disregard capital due to the short-run nature of his model is quite restrictive. It is true that the stock of capital will not change to a very great extent, but there is no reason to believe that the price of capital cannot vary to a great extent. In the static model being presented, the consumer can receive a wealth effect on all assets he possesses; the price of capital is assumed to be independent of the price of consumption goods.

Finally, if P_L rises relative to P_C, the real income of the consumer has increased, and again he will undertake more present consumption. Thus, in a static model, there are three resources at the consumer's disposal, the sum of which constitutes his budget constraint. The first is his initial net worth. The second is the change in real value of his net worth due to the change of P_M and P_K relative to P_C, a change which will be called capital gains. Finally, there is the third resource, real income, which is determined by the wage rate, P_L, relative to P_C. Nevertheless, even in a short-term static model, the price of capital relative to the price of consumption goods may change, thereby giving a real capital gain or loss on the stock of capital goods. A more comprehensive definition of the Pigou or wealth effect should include such gains on capital as well as on monetary assets.

(In some of the literature there appears to be some confusion as to whether Patinkin meant to focus on the "one-shot" affair, i.e., upon the

---

1. Don Patinkin, Money, Interest and Prices (New York: Row, Peterson and Company, 1957), p. 126.

2. Ibid., p. 126, n. 2.

3. Ibid., p. 130.

4. An interesting aspect of this capital gains problem emerges when one includes the possibility of such gains from capital goods. With Pigou and Patinkin, a real capital gain on the part of the private sector meant a corresponding capital loss for the government sector. Therefore, these capital gains were merely a redistribution of wealth from the government sector to the private sector. For the economy as a whole (assuming no net foreign debt) there was no real net gain or loss.

Now, however, such a conclusion is not evident. If the value of capital goods rises relative to that of consumer goods, there is a real capital gain, but no corresponding capital loss. As capital gains have been defined, the economy as a whole is now wealthier.

capital gain, or rather upon the permanent effect which repeats itself period after period, namely the initial wealth effect.[1] To avoid confusion in the matter both the flow and stock variables will be included in the consumption function set forth at the end of this section; but it seems clear from the form of the Patinkin consumption function above that he definitely had the stock in mind. Furthermore, there is not a precise definition of the time period involved; thus, this separation of the flow from the stock is not significant because the flow adds to the stock, and one could make the period short enough to focus only on the initial stock.)

One interesting implication of this static model is that all three resources of the budget constraint are perfect substitutes for one another. Any equal increase in the value of resources will result in the same equilibrium position regarding consumption goods. (This conclusion is true only of the two-period model, but not necessarily true in the models of longer time horizon, a point which will be considered below.) Tobin makes allusions to this conclusion in the excerpt quoted in the Preface.

Two-Period Model of Consumption

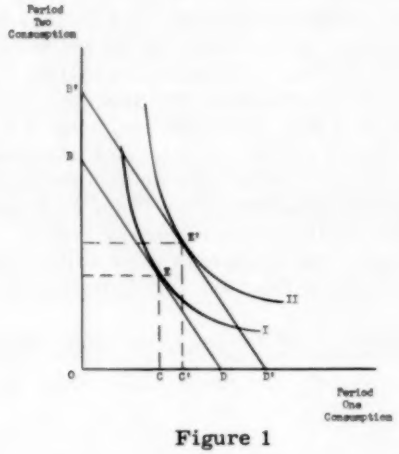


Figure 1

1. For example, Modigliani and Brumberg consider him to mean only the capital gain component. "Just as we do not expect an increase in income lasting but one period to raise consumption permanently to a new level, so we cannot expect a once-and-for-all fall in prices to reduce saving permanently. In other words, the 'Pigou effect' wears off." "Utility Analysis and Aggregate Consumption Functions: An Attempt At Integration," op. cit., p. 39.

Given the interest rate, the higher the budget line BD (Figure 1), the larger the quantity of consumption goods desired.[1] But BD can be shifted by varying any of the resources. Suppose BD shifts to B' D' because of an increase in income (P_L rises). The consumer then moves to E' where consumption rises to OC'. Saving also rises from CD to C' D' since it is a normal good. This case exemplifies the classic textbook treatment of an income change and time preference. If, however, the consumer receives additional resources of the same size, DD', from capital gains due to price level changes rather than from increased income, he will still be in equilibrium at E'. (Saving has now declined from CD to C' D.)

Furthermore, suppose the consumer had initial wealth of DD' and an income still equal to OD (or this could be interpreted as the second period where the consumer had saving of the first period equal to DD'). The same solution at E' will be obtained, with saving falling from CD to C' D. (Actually, the income variable is flexible in this model since theoretically the consumer can trade off as much leisure as he wishes at the given P_L; but this difficulty is averted by assuming that the amount of leisure traded off for income in any period is fixed by tradition and that the individual cannot vary the trade by any substantial amount, i.e., the work week or period is more or less given. Therefore, the consumer cannot determine income but can only dispose of it as he sees fit. Income is an exogenous variable.) Nevertheless, the result is apparent; any increase in resources of a given amount, regardless of source, leads to the same equilibrium position in consumer choice. Net worth at the end of the period is the same in all cases.

B. Modigliani and Brumberg Model

The above model, while defining the Pigou effect, is quite narrow and static in its time horizon. Current theories of consumption are built around a longer time horizon[2] which views consumption over the lifetime of the individual. This lifetime consists of many finite equal time periods, and consumption is considered in this broader context rather than in a one- or two-period model.

1. The budget line BD represents the capitalized income stream, and consequently income is a stock in this analysis. In Period 1, the maximum potential resources, OD, would be the income of Period 1, Y_1, plus the discounted value of Period 2 income, i.e., $Y_2 / (1 + r)$, where r is the interest rate at which the consumer can lend or borrow. In Period 2, maximum potential resources, OB, would be Period 2 income, Y_2, plus the income of Period 1, Y_1, and the interest earned on this income, rY_1. The line BD is the resources constraint line with both period incomes as the only resources. Such incomes are treated as a stock.

However, to simplify the analysis, OD is treated as if it were all Period 1 income. There is no income in Period 2. This simplification does not affect the results of the model, but would alter slightly the definition of saving.

2. For example see Milton Friedman, A Theory of the Consumption Function (Princeton, New Jersey: Princeton University Press, 1957).

In the Modigliani and Brumberg model the consumer derives utility from the consumption of goods, and he must also allocate consumption over time as well as among goods. Furthermore, he derives utility from the amount of wealth (accumulation goal) he has at the end of his lifetime to pass on to future generations. (There is a similarity between one-period consumption and saving and lifetime consumption and the accumulation goal.) Thus the arguments of the utility function are period consumptions and ending wealth, i.e., $U = U(C_t, C_{t+1}, C_{t+2}, \ldots, C_E; W_E)$, where E is the last period of economic activity.

In the basic system of Modigliani and Brumberg,[1] simple assumptions are made about both initial wealth and final target wealth, namely, that both are zero. They thereby constrain the individual to consuming over the planning period (his economic lifetime) all of the resources, both income and windfall gains, received during the period. Furthermore, they assume the interest rate to be zero.

In Figure 2, the consumer allocates consumption equally over the planning period. He earns income until time N, at which time he retires and lives until time E. Until time N he has positive saving; his wealth grows. After N he dissaves, exhausting his wealth. He consumes equal amounts each period over the whole time span. Figure 3 shows the effect of a windfall, "one-shot", capital gain which is unanticipated. (It should be noted that consumption rises with the receipt of the gain, and the gain is spread over the remaining life.) Figure 4 shows the effect of a rise in permanent income. Again consumption rises. To provide for a target bequest in the model, W_E, a lower rate of consumption is maintained as in Figure 5.

This is a simple model which assumes that the interest rate is zero; that pure time preference exists; that income is equal each year; and that perfect certainty of the life span exists. But it does show the impacts of capital gains upon consumption; it does show how net worth varies over the life cycle; and primarily it relates consumption not to a static one- or two-period model, but rather to a planning period of the entire economic lifetime. The model also puts all constraints in real terms.

It should be noted that the changes in income or in windfall gains were completely unanticipated. If a change in total resources was fully anticipated, e.g., a gain, then consumption would have been fully adjusted to the increase throughout the entire time period and not just at time R. This result is the same as that of Friedman who claimed that such increases, if fully anticipated, would already have been incorporated into the consumption pattern. The contrast is quite evident from the comparison of Figure 3 with Figure 6. If the gain were of sufficient size, saving could be negative in the early years.

There is a sharp contrast here with the static model. An increase in resources, whether from income or from capital gains in the one- or two-period model, would increase present consumption by the same amount, i.e.,

1. Much of the following analysis was derived from lectures given by Professors James Tobin and Arthur M. Okun at Yale University, 1960–61.

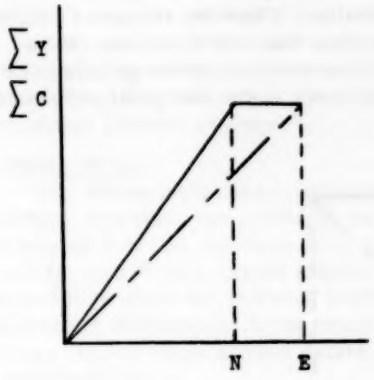


Figure 2

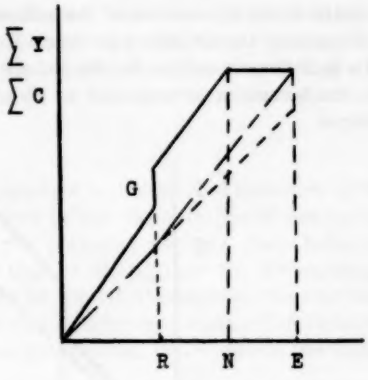


Figure 3

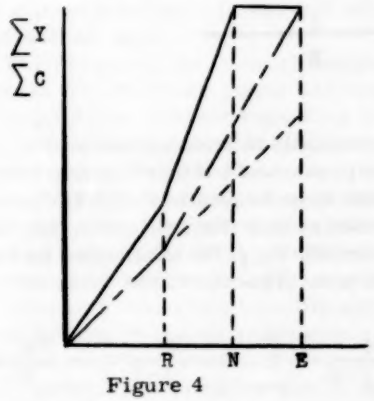


Figure 4

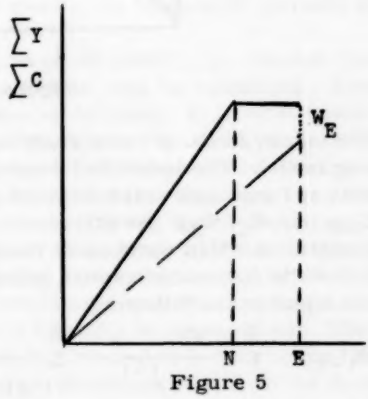


Figure 5

Total Consumption——— – ——— – ——— – ——— – ——— – ——— – ———
Total Resources ———————————————————————————————
Total Net Worth or Accumulated Savings is the vertical distance between two lines.

for each resource the consumer would have the same marginal propensity to consume. The source of the increase is irrelevant. However, the marginal propensities to consume are not the same in the multi-period model.[1] In

1. If the person were in the last two periods of the longer model, the impact upon consumption of any resource increase would be the same as in the

Modigliani and Brumberg's model the current income flow is expected to repeat itself time after time (Figure 4). The capital gain, however, is a "one-shot" affair to be spread over the whole lifetime. Thus the marginal propensity to consume out of gains is much lower than that out of income. It is lower in direct proportion to the length of time over which the gain is to be spread; the longer a person has to live, the lower is the marginal propensity to consume.

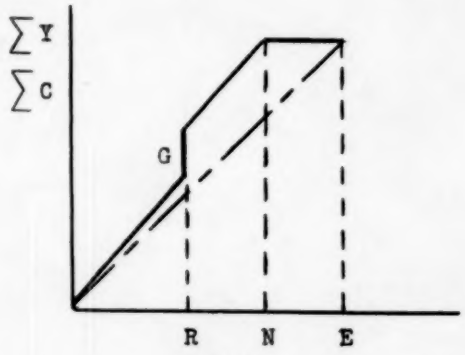


Figure 6

The model in the present study is essentially this Modigliani and Brumberg model. The individual consumer possesses a utility function for real goods and services which includes goods to be bequeathed; $U = U(C_t, C_{t+1}, C_{t+2}, \ldots, C_E; W_E)$. He strives to maximize this function, given the budget constraint which consists of initial wealth, W_{t-1}, the discounted income stream, and the discounted capital gains stream. Therefore, the consumer behaves subject to the following:

$$W_{t-1} + \sum_{f=t}^{N} \frac{Y_f}{(1+r)^{f-t}} + \sum_{f=t}^{E} \frac{G_f}{(1+r)^{f-t}} = \sum_{f=t}^{E} \frac{C_f}{(1+r)^{f-t}} + \frac{W_E}{(1+r)^{E-t}}$$

where N is the year of retirement and E is the year of death. (N is less than E and E−N is the retirement span.) The interest rate, r, is the rate in each period, f; however, for the consumer in this model, r is to be interpreted as an "average rate" of the expected rates of interest over the life span, so that the same rate is expected each period.

---

short-period model, i.e., the one- or two-period model is essentially the same as the last part of the Modigliani and Brumberg model (ignoring the absence of income in this retirement span of their model). The short-period model, therefore, could be interpreted as a part of the longer one; but for a more complete analysis, the longer-period model should be used.

This is basically the Modigliani and Brumberg model with the inclusion of capital gains as an explicit variable. The receipt of such gains will result both in more consumption and in a higher bequest or accumulation goal (assuming both goods are normal). Effectively the budget constraint or value of total resources increases with such gains thereby permitting a higher point on the utility function. All behavior is in terms of real variables; there is no money illusion present.[1]

C. Basic Model

The framework presented above applies to the entire lifetime of the consumer; however, the principal objective is the explanation of the consumer's behavior in any one time period of the planning period. Such behavior can be derived from a model similar to that of Modigliani and Brumberg.[2] There always exist net positive additions to wealth throughout the earning span and net decrements from wealth in the retirement span. Therefore, at any time period there always exists a target wealth, W_t^*,[3] which the individual

1. It is assumed that the time horizon or life span of the model is broken into small, equal, finite time periods, each of which has its own interest rate, price level structure, flows, and initial stocks. In Section III, periods of one year will be used.

Furthermore, the internal composition of net worth, i.e., its distribution between net monetary assets and capital assets, will be examined. Such a study might be relevant regarding the saving decision. It is often asserted that the higher the level of monetary assets relative to capital assets in the possession of the individual, i.e., the more liquid his portfolio, the lower his propensity to save. Studies undertaken have revealed that liquid asset holdings have a positive effect upon consumption. (These liquid assets are often used as a proxy for total assets.) However, this study will assert that the degree of liquidity over a planning period is under the control of the individual consumer, and while liquidity and consumption may be correlated, there is no necessary causal relationship from liquidity to consumption. The individual may have planned the consumption first and then have made his portfolio more liquid to enable him to make his purchase. Thus, in the dynamic model, the consumer would be influenced by the total value of net worth and not by the internal composition of assets. As long as any individual is free to convert his assets from one type to another at relatively low cost, the internal composition of net worth should bear no relation to consumption in this model. This hypothesis will be tested in Section III.

2. It should be noted that the model to be presented is one of many that are consistent with the general framework of the Modigliani and Brumberg model outlined above.

3. Wealth, W_t, is that value of net worth at the end of the period t. Any variable with an "*" superscript is a target or desired value of the variable. Any variable with an "e" superscript is the expected or anticipated value of the variable. Any variable with no superscript is the actual value of the variable.

hopes to achieve at the end of period t. If period t is in the earning span, $W_t^* > W_{t-1}^* > W_{t-2}^* \ldots$ etc. If t and t–1 are in the retirement span, $W_t^* < W_{t-1}^*$. (See Figures 2 through 5).

The relevant comparison for the consumer in this model is the comparison between the target wealth at the end of the period, W_t^* actual wealth at the beginning of the period, W_{t-1}, and expected capital gains of the period, G_t^e. The relevant behavioral assumption is that consumption, C_t, or more precisely saving, S_t, is a function of the gap between desired ending wealth and potential ending wealth, which consists of initial wealth and the change in initial wealth due to price level movements, i.e.,

$$(1)\quad S_t \equiv \theta[W_t^* - W_{t-1} - G_t^e]$$

subject to the following definitional equations. Saving is the difference between income, Y_t, and consumption, C_t, i.e.,

$$(2)\quad S_t \equiv Y_t - C_t$$

Wealth can change from one period to the next either by saving or by capital gains, G_t, i.e.,

$$(3)\quad W_t \equiv W_{t-1} + S_t + G_t$$

The real issue then becomes the explanation of the factors that determine the target value of wealth, W_t^*. (In the Modigliani and Brumberg model the resource variables are clearly the primary factors.) Stated in a general form, target wealth would be $W_t^* = h(Y_t^e, G_t^e)$, which has been placed in the following linear form:

$$(4)\quad W_t^* = \gamma_1 + \gamma_2 Y_t^e + \gamma_3 G_t^e + \gamma_4 A_t$$

where A_t is the age of the consumer. (Desired wealth will be a function of the position of the consumer in the planning period. This factor becomes important in an aggregate function where age distribution could affect aggregate consumption depending on how many persons are in the retirement or dissaving span. See Section II for further discussion.) Both G_t^e and Y_t^e are expected values, each of which should be interpreted as the average of the discounted flow of the variable. (Both expected values are related to the current values, as set forth below.)

Furthermore, by making very simple assumptions about the expected value of income and capital gains, a consumption function is derived for the period t. Let the expected value be a linear function of the actual value,[1] i.e., expected value be a linear function of the actual value,[1] i.e.,

1. While the assumption that expected and current values are linearly related may be weak when applied to the individual consumer, such an assumption may be valid at the aggregate level, and it is at this level of behavior that the present study is focused. Income received in the aggregate probably will not differ greatly from that anticipated in the aggregate, and any differences could be accounted for by the constant term λ_1, and the coefficient λ_2.

$$(5)\quad Y_t^e = \lambda_1 + \lambda_2 Y_t$$

$$(6)\quad G_t^e = \phi_1 + \phi_2 G_t$$

Then one derives the following:

$$(7a)\quad S_t = \delta_1 + \delta_2 Y_t + \delta_3 W_{t-1} + \delta_4 G_t + \delta_5 A_t$$

$$(7b)\quad C_t = B_1 + B_2 Y_t + B_3 W_{t-1} + B_4 G_t + B_5 A_t$$

where

$$\delta_1 = \theta\gamma_1 + \theta\gamma_2\lambda_1 + \theta(\gamma - 1)\phi_1 \qquad B_1 = -\delta_1$$

$$\delta_2 = \theta\gamma_2\lambda_2 \qquad B_2 = (1 - \delta_2)$$

$$\delta_3 = -\theta \qquad B_3 = -\delta_3 = \theta$$

$$\delta_4 = \theta(\gamma_3 - 1)\phi_2 \qquad B_4 = -\delta_4$$

$$\delta_5 = \theta\gamma_4 \qquad B_5 = -\delta_5$$

Consumption is therefore a function of the three resource variables, Y_t, G_t, and W_{t-1}; and of age, A_t.

This consumption function can be modified by changing the prior equations from which it is derived. For example, lagged income can be readily introduced into the consumption function by altering Equation (5) so that expected income is a function not only of current income but also of lagged income. Similarly, lagged capital gains can be introduced into the consumption function by altering Equation (6) so that expected capital gains is a function not only of current capital gains but also of lagged capital gains. In this part of the study only the basic model will be considered.

Furthermore, there is the possibility of a cyclical consumption model being derived from Equation (1). Similar work has been done regarding capital formation and inventory holdings; however, it is the intention of this thesis not to examine the theoretical and mathematical relationships of the specific model but rather to examine the empirical effects of the broader Pigou effect upon consumption. Therefore, further theoretical detail will be omitted.

However, there are some basic properties that should be presented. First of all, this model can imply that a rise in expected income or expected capital gains will increase current consumption, i.e.,

---

Similarly, one could assert that expected capital gains for a period will not differ greatly from those actually received, and furthermore, the specific form of the equation gives sufficient flexibility to the model to allow for a variety of assumptions about capital gains. If the consumer expects prices to be stable, then he anticipates zero capital gains ($\phi_1 = 0$; $\phi_2 = 0$); if he expects prices to change so that he receives capital gains of a constant amount, k, then $\phi_1 = k$; $\phi_2 = 0$; if he expects prices to change so that he anticipates capital gains equal to those actually received, then $\phi_1 = 0$; $\phi_2 = 1$; etc.

$$\frac{\partial C_t}{\partial G_t^e} > 0 \text{ implies } \frac{\partial C_t}{\partial G_t} > 0 \text{ implies } B_4 > 0$$

$$\frac{\partial C_t}{\partial Y_t^e} > 0 \text{ implies } \frac{\partial C_t}{\partial Y_t} > 0 \text{ implies } B_2 > 0$$

(These two conditions are true, however, only if other conditions are satisfied within the model:

1. In the consumption function the marginal propensity to consume out of income, B_2, is assumed to be positive and less than one,

$$0 < B_2 < 1$$
$$0 < 1 - \gamma_2\lambda_2\theta < 1$$

or $\gamma_2\lambda_2\theta > 0$

but (from below) $\theta > 0$

therefore $\gamma_2\lambda_2 > 0$

but (from below) $\gamma_2 > 0$

therefore $\lambda_2 > 0$

Thus, the model must assume that part of current income received is anticipated.

2. In the consumption function the marginal propensity to consume out of capital gains is assumed to be positive, i.e., $B_4 > 0$.

$$B_4 > 0$$
$$\theta(1 - \gamma_3)\phi_2 > 0$$

but (from below) $\theta > 0$

therefore $(1 - \gamma_3)\phi_2 > 0$

but (from below) $\gamma_3 > 0$

if $0 < \gamma_3 < 1$, then $\phi_2 > 0$

if $\gamma_3 > 1$, then $\phi_2 < 0$

Since received capital gains may be partially anticipated, the model prefers the assumption that ϕ_2 is positive; therefore, the model assumes that $0 < \gamma_3 < 1$

Second, the wider the gap between desired and potential wealth, the higher should be current saving,

$$\frac{\partial S_t}{\partial (W_t^* - W_{t-1} - G_t^e)} > 0 \text{ implies } \theta > 0$$

or

$$\frac{\partial C_t}{\partial (W_t^* - W_{t-1} - G_t^e)} < 0$$

As a corollary to this second property, if W_t^* and G_t^e are given, then clearly, the larger W_{t-1}, the greater is consumption, i.e.,

$$\frac{\partial C_t}{\partial W_{t-1}} > 0 \quad \text{implies} \quad B_3 > 0 \ .$$

Third, W_E^* or ending wealth is assumed to be a normal good so that

$$\frac{\partial W_E^*}{\partial Y_t^e} > 0$$

$$\frac{\partial W_E^*}{\partial G_t^e} > 0$$

Also

$$\frac{\partial W_t^*}{\partial Y_t^e} > 0 \qquad \text{implies} \quad \gamma_2 > 0$$

$$\frac{\partial W_t^*}{\partial G_t^e} > 0 \qquad \text{implies} \quad \gamma_3 > 0$$

Thus the above model presents a consumption function which implies that the more you have of any of the three resource variables in any period, the higher will be consumption. Among the resource variables being considered is the Pigou or wealth effect.

D. Summary of the Model

In algebraic terms the following is the basic micro model being examined.

$$U = U(C_t, C_{t+1}, \ldots, C_E; W_E)$$

$$W_{t-1} + \sum_{f=t}^{N} \frac{Y_f}{(1+r)^{f-t}} + \sum_{f=t}^{N} \frac{G_f}{(1+r)^{f-t}} = \sum_{f=t}^{N} \frac{C_f}{(1+r)^{f-t}} + \frac{W_E}{(1+r)^{E-t}}$$

$$S_t = \theta[W_t^* - W_{t-1} - G_t^e]$$

$$W_t^* = \gamma_1 + \gamma_2 Y_t^e + \gamma_3 G_t^e + \gamma_4 A_t$$

$$G_t^e = \phi_1 + \phi_2 G_t$$

$$Y_t^e = \lambda_1 + \lambda_2 Y_t$$

$$Y_t \equiv C_t + S_t$$

$$W_t \equiv W_{t-1} + S_t + G_t$$

Therefore, $C_t = B_1 + B_2 Y_t + B_3 W_{t-1} + B_4 G_t + B_5 A_t$ where B_2, B_3, B_4, γ_2, γ_3, and θ are all believed to be positive.

SECTION II

Conceptual Problems: Definitions and Aggregation

In the preceding section a consumption model of the individual was set forth which incorporated the Pigou effect. This section will discuss some of

the definitional and theoretical difficulties involved in the micro model and will then proceed to the problems of aggregation.

A. Definitional and Conceptual Problems

The first variable that should be defined is consumption itself. What is meant by this term; what factors does it contain? Second, when reference is made to a consuming unit, what precisely is meant by this term; an individual, a household, a family? Involved in these definitional concepts is a decision process by the consuming unit concerning factors over which it has some control.

In simplest terms consumption is viewed as the goods and services actually "used up" by the consuming unit, i.e., it is some flow per unit of time. Viewed in the dynamic model, it is a flow over the individual units of time in the planning period.

This definition tends to be theoretical since only the use of such goods and services is consumption. An inventory of goods that are for future consumption is not considered consumption until actually "used up." Such an inventory would include consumer durables whose services extend beyond one time period. Thus the "ideal" definition of consumption would be the service of the good as it is actually used up. Any acquisition of such a good is only a reshuffling of the consumer's wealth or portfolio. The purchase is not consumption; it is merely a balance sheet change.[1]

For example, the purchase of an automobile for $2,000 means a reduction of cash by $2,000 and an addition to durable assets of a car worth $2,000. Net change is zero. (While the internal composition of assets may be related to consumption, as discussed above, the model under consideration is not examining the mix but is focusing upon the effect on consumption of total assets—more precisely, net worth.)

When the automobile is used, however, consumption is occurring; the problem is to impute the "correct" amount. Therefore, some charge against the car's value must be made to record the consumption. Unfortunately, the amount to impute is not well-defined since several alternatives exist. The first possibility is to impute an "income" from the good along the lines followed in imputing a rental income to owner-occupied homes.[2] On the other side of the account, there should be a charge for imputed consumption attributed to the use of the good. Since these two sides are equal (assuming a double-entry bookkeeping procedure), there is a marginal propensity to consume of one. Furthermore, as an additonal expense, there is the depreciation of the durable to be considered. This is an expense or deduction on one side of the income statement and negative saving on the other side (see

1. For an early discussion of this subject, see Irving Fisher, The Nature of Capital and Income (London: Macmillan Company, 1906).

2. United States Department of Commerce, National Income, A Supplement to the Survey of Current Business (1954 edition; Washington, D.C.: U.S. Government Printing Office), p. 46.

Figure 7a). The net effect of using this durable on the balance sheet is to reduce the asset and net worth value by the amount of the depreciation (see Figure 7b). On the flow statement, income and consumption have increased equally; depreciation is an additional charge.[1]

Income Statement

Expenditure		Income	
Consumption	$110	Rental Income	$110
Depreciation	100		
Total	$210		$110

(Saving = $110 − $210 = $− 100

Figure 7a

Balance Sheet Changes

Assets			Liabilities and Net Worth	
	Cash	−$2000		
Automobile	+$2000			
Less: Reserve for Depreciation	−100		Net Worth	−$100
		+1900		
Net Change		−$ 100		−$100

Figure 7b

Such treatment as outlined above is attempted when dealing with owner-occupied homes in the national accounts. For consistency, similar treatment should be extended to all durables.

If such an imputation is attempted, however, two additional problems arise. First of all, what income or rental value should be placed on the durable; and second, how much depreciation should be charged against it? (In Figure 7 all charges were selected arbitrarily for convenience.) "Correct" depreciation is an area of much uncertainty in itself. Which method is correct:

1. Gross income is $110; net income is $10 (gross income of $110 less depreciation of $100); saving is $−100 (net income of $10 less consumption of $110). Consequently net worth declines by the amount of dissaving, $100, due to this transaction for the period.

original cost, straight line, sum-of-year's digits? This concept is further complicated by price-level changes which bring one into possibilities of writing off more or less than the original cost, depending on whether replacement cost depreciation is being used and on the direction in which prices are moving. This factor is exceedingly important when price-changes result in capital gains from the possession of such goods, especially since capital gains are an important resource variable in the dynamic model. Also, there is the price index problem which includes the possibilities of quality changes and the resulting difficulty in adjusting for them. There is no simple clear-cut amount to charge for depreciation.

The second problem is that of imputed rents. The depreciation figure alone cannot be used as the "correct" amount of rent. There does exist an opportunity cost as in capital theory. Instead of purchasing a car, the individual could have purchased a security which would yield a return. Thus the imputed rent must include not only a write-off of original investment but also an imputed interest return. What is the correct interest return? (In the dynamic model there is only one rate to lend at or to borrow at, thus most of this difficulty is bypassed.) Furthermore, there may be the need for an additional imputation due to a risk element in Knightian terms. This could be a third factor to consider in imputing a rent.

If a perfect market existed then such rents as determined in the market could be used; however, there is no rental market for most consumer durables. Good market values do not exist for items such as the rent of an electric toaster. (For items such as a house or possibly an automobile, an approximation could be made; however, any such imputed value will involve much arbitrary choice and the problem is still not fully solved.) Thus the area of imputed rent is unclear.

Due to these two elements, depreciation and imputed rent, the term consumption becomes quite blurred when one tries to adhere to a theoretically pure definition of goods and services used up, especially if durables are brought into the picture. Both conceptually and in a practical sense, such imputations are weak at best. Consequently, both as a matter of convenience and as a matter of quantitative significance, the definition of consumption being used in this study will be the definition of the Department of Commerce less net (of replacement cost depreciation) accumulation of consumer durables. Such a definition properly considers home construction and durables as a part of saving.

The income variable is another flow to consider. Several possible definitions could be used, but since this is a model of decision-making by the consumer, disposable income is the preferable definition. It is this income which the consumer has the greatest freedom to allocate as he desires. Taxes may vary, changing the net amount under the individual's control even though gross income is the same. Disposable income changes but gross income does not; thus disposable income is the preferred concept in this context. For convenience this will include net transfers, otherwise such transfers would have to be considered as a fourth element of total resources.

As a part of this income, the imputed income of all durables and owner-occupied houses should be included, as discussed above. (A similar amount should be included in consumption.) This step is necessary to be logically consistent and complete in dealing with durable goods. Again, however, the same imputation problems arise. The "correct" income variable is not a simple, obvious choice. Consequently, because of this imputation difficulty, no addition for imputed income from durables is included, even though depreciation of such durables is a part of consumption. This study uses the definition of disposable income of the Department of Commerce (which does contain an imputed rental income from owner-occupied homes) except that depreciation is on a replacement-cost basis and not upon an original-cost basis. See the Appendix for a detailed analysis and sources of the data.

(Such a definition of income may seem inconsistent with the above discussion; however, one of the empirical tests was conducted with a definition of disposable income that included the depreciation value as the imputed income of durables. The statistical results are essentially the same as those that did not include this imputation. The use of this definition of income was, therefore, discontinued since the imputation is quite arbitrary and the results give no additional information.)

Finally, there is the problem of the consuming unit itself. Must such a unit be considered; and if so, what is it? To facilitate the discussion somewhat, the consuming unit is assumed to exist. If the consuming unit is necessary, then the problem of choosing or defining the "correct" unit will be considered.

From the theory of the micro model, it is seen that the inclusion of such a unit is necessary. The micro model is a decision model; some unit makes this decision. Furthermore, in the aggregation process, a specific allowance for this factor must be made.

It is plausible to assume that there exists a consumption function that is curvilinear or that has a positive intercept so that the marginal propensity to consume is less than the average propensity to consume. If this situation exists, account must be taken of the number of consuming units. The use of only aggregate gross observations is insufficient because differing values of consumption are possible at the same income level. (If the marginal propensity to consume equals average propensity to consume—e.g., a linear function with no intercept—then it makes no difference whether or not such a unit is taken into account, but the inclusion of such a measure will significantly help the analysis if the marginal propensity to consume and the average propensity to consume differ.)

If income remains the same over time, but the number of consuming units rises (say, as population rises), current aggregate consumption would be expected to rise. If not, this would imply that real consumption (or the living standard) per unit is falling. If there exists a function relating consumption per unit to income per unit as in the following diagram (Figure 8), and if population rose, then at the same total income there would be a movement from point A to point B. But point B is one of higher average propensity

to consume, and therefore total consumption rises in the aggregate. If only total income and total consumption are examined over time, with no allowance for the rising population or number of units, the consumption function would appear to be shifting to the left, while the per unit function would appear stable. Thus it is necessary to use variables which are exclusively on a per unit basis, especially for an aggregative study over time.

Consider the following examples. If each unit's consumption function were

$$c = b + a(Y/n)$$

where Y equals aggregate income and n is the number of consuming units, then aggregate consumption would be

$$(1)\quad C = nb + aY \;;\; \text{where } C = nc$$

Now, if the population doubled, at the same Y, then aggregate consumption, C', would be

$$(2)\quad C' = 2nb + aY$$

Clearly,

$$C \neq C' \;;\; C' > C.$$

Even though gross income is the same, consumption has risen.

However, if both n and Y doubled, then aggregate consumption, C'', would be twice original consumption.

$$(3)\quad C'' = 2nb + 2aY \text{ or}$$

$$2C = C''$$

Conversely, if only Y doubled, aggregate consumption C''' would be

$$C''' = nb + 2aY$$

$$(4)\quad C''' \neq 2C$$

$$C''' < 2C = C''$$

Thus, by using only the gross figures of C and Y, differing aggregate consumption values will be obtained if the change in the number of consuming units is disregarded. In cases (3) and (4), income doubled but the final consumption figures were not the same. In cases (1) and (2), gross income was the same, but final consumption again varied. Thus it would appear that allowance must

be made for the factor of the consuming unit. The question now is: What is the "correct" consuming unit?

As discussed thus far, this unit has been assumed to be an individual. Such a practice is quite often correct, as in the case of a person living alone. However, in the case of a married couple, it is clear that each person is not an independent consumer in the same sense that each would be if he or she lived alone. To a large degree, they possess a joint utility function since there exists an interdependency between the two persons regarding preferences, as well as a common budget constraint. As the household grows in number, the individual becomes an even poorer choice for the consuming unit, perhaps making the family a better choice. However, this conclusion is not a perfect one either. The family is not necessarily the best unit. A one-person family with $2,000 of income will not consume the same amount as a two-person family with $2,000 of income (tastes and other factors equal). Under these circumstances, as the number of persons increases, consumption will increase.

Per Unit Consumption Function

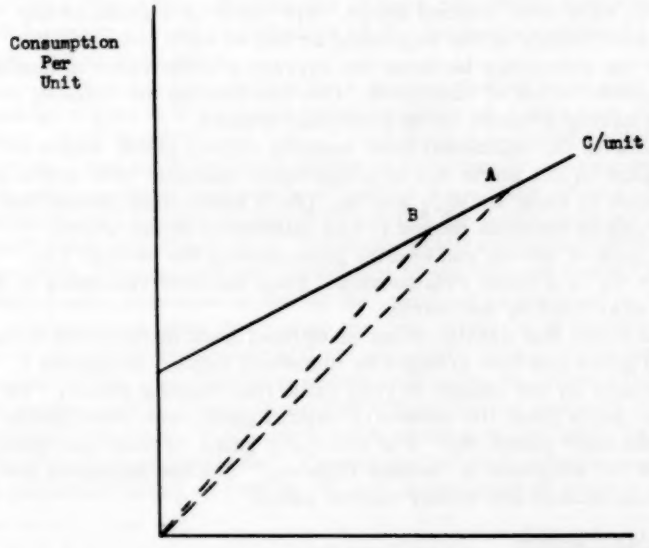


Figure 8

Conversely, even though the family is not the correct unit, it cannot necessarily be concluded that the individual is the correct consuming unit. A two-person family with $5,000 will not consume the same proportion of income as a four-person family with $10,000. The two-person family will

consume proportionally more (there appears to be an economy on some goods that are more fixed in nature, such as rent), yet on an income per head basis, the families are equal ($2,500 per person).

The answer to the question of the correct unit appears to have no clear-cut solution. The "correct" consuming unit is neither the individual nor the family. These cases can be made more complex by adding other factors, such as working children or paying boarders, but the point remains clear that there is no unit which is the unequivocal choice under all circumstances. However, in time-series studies, if one considers consumption on a per unit basis, the use of the individual or family as the unit will make little difference statistically since both measures are highly correlated with one another. The increasing numbers of units must be taken into account, however, as discussed above. This study prefers to examine consumption on a per person basis since the definition of a family or household is somewhat arbitrary, while population is a clearly defined concept.

Other key variables that are included in the dynamic model, and which should be defined, are net worth, saving, and capital gains. These variables are linked to one another over the time periods. Saving is a flow and is viewed as that portion of income not consumed, i.e., $S_t \equiv Y_t - C_t$; income Y_t and consumption C_t have been defined above. Net worth is a stock at any moment of time, specifically at the beginning or end of each time period; it is defined to be the difference between the current market value of assets and the current market value of liabilities. One can change the value of net worth through the saving process or by price movements.

Finally, there is the additional flow, namely capital gains, which is defined as the change in net worth due to price-level changes. Net worth is denoted at two points in time, by W_{t-1} and W_t. (Such subscripts denote the value of W at the end of the time period.) The difference in net worth, $W_t - W_{t-1}$, is the sum of saving and capital gains during the period, i.e., $W_t \equiv W_{t-1} + S_t + G_t$ is a basic relationship. Thus the flow variables in the model are linked over time by net worth.

It should be noted that capital gains as defined here differ from those of Section I. Such gains are now changes in monetary value. In Section I, the discussion focused on the change of real value (purchasing power). To derive real capital gains from the monetary capital gains, one must divide by the price of consumer goods, P_C. The monetary gains versus real gains can be used to test for evidence of "money illusion." See the Appendix for a complete discussion of real and money capital gains.

B. Problems of Aggregation

The discussion thus far has focused primarily upon the dynamic model and on definitions in reference to the consuming unit. However, both for forecasting purposes and for policy decisions, an aggregative model is usually preferred. In moving from the micro level to the aggregate level, the well-known aggregation and index-number problems arise. The basic decision-maker is still the individual consuming unit, but now difficulties occur in aggregating such decisions.

First of all, there is the problem of the distribution of the resource variables and the weight to be given to them in the so-called aggregate consumption function. Ideally, the aggregate function could be derived by adding up the individual units and by weighting the observations by the size of their resource factors. For example, by assuming a micro and an aggregate consumption function of the following form:

$$C_t = B_1 + B_2Y_t + B_3W_{t-1} + B_4G_t,$$

individual observations could be added into an aggregate function.[1] But if the individual B's vary, the aggregate B's will vary with the distribution of the factor in question. Out of a given amount of Y_t, aggregate C_t will vary depending on Y_t's internal distribution. Somehow the individual B's must be weighted to obtain the B's that would be observed in the aggregate measure. A shift of income from a group with lower marginal propensity to consume out of income (MPC_y) to a group of higher MPC_y would raise aggregate C_t even though the aggregate values of Y_t, G_t, and W_{t-1} remained the same. Similar results will also occur regarding a shift of G_t or W_{t-1} between groups with differing MPC's. Thus, for work with aggregative material, particularly over time where the distributions of such resources may change, allowance must be made for the distribution of the resource variables: net worth, income, and capital gains.

Empirical, aggregative time-series studies have been performed which incorporate a one-parameter variable as a measure of income distribution. Both Staehle[2] and Polak[3] used such a variable in their regression equation. The primary difficulty is to conceive of a measure that is a one-parameter variable and which will allow for all the effects of distribution. Furthermore, empirical observations for the variable must be available. Both Staehle and Polak used measures which attempted to evaluate the concentration of income in the upper part of the income scale. The reasoning behind such a measure is that if the marginal propensity to consume falls as income rises, then the higher the concentration of income at the upper end of the scale, the lower will be aggregate consumption out of any level of income. (This distribution problem applies to the other variables of wealth and capital gains. There should be a distribution measure for each resource variable. However, there were little or no data available regarding the distribution of these variables. Consequently, such variables were not used in the empirical study of Section III; but, a measure of income distribution was

1. For a thorough discussion of this problem, see Fritz C. Holte, Economic Shock-Models (Norway: Norwegian Universities Press, 1961), pp. 75–116.

2. Hans Staehle, "Short-Period Variations in the Distribution of Income," Review of Economic Statistics, Vol. 19 (1937), pp. 133–43.

3. Jacques J. Polak, "Fluctuations in United States Consumption, 1919–1932," Review of Economic Statistics, Vol. 21 (1939), pp. 1–12.

used and it is probably reasonable to assume that high income recipients, large wealth holders, and those who receive capital gains are roughly the same. Therefore, the distributions of such resources may be highly correlated with one another, and the inclusion of only one distribution measure may be sufficient.) Both Polak and Staehle found that the inclusion of such a variable in the regression equation significantly increased the R^2 (coefficient of determination).

The one-parameter measure of income distribution used in this study is the degree of income concentration at the upper part of the income distribution. Like the Polak and Staehle measures, this measure is based upon the assumption that the marginal propensity to consume is lower at the higher income levels. The measure is the per cent of disposable income received by the upper five per cent of income recipients. Cross-section studies imply that only the top income levels have lower MPC's; consequently, this measure of distribution is used in the aggregate consumption function to allow for shifts in the internal distribution of total income.

One other factor to consider in an aggregate function is the age distribution of the population. In the theoretical model of this study, persons in the earnings span have positive saving; those in the retirement span have negative saving. If over time the population shifts from one span to the other, aggregate saving should vary. As a larger proportion of the population reaches retirement, aggregate saving should fall at any given level of income.[1] Since 1900, the per cent of population in the retirement span (persons over 65) has doubled; this change should have resulted in a leftward shift of the consumption function.

C. Conclusion

What are the conclusions of this chapter? It seems quite evident that in such an examination of aggregate saving, the saving concept should include the net accumulation of durables, and the definition of consumption should be the goods and services actually consumed. Furthermore, the income variable is that income which the consumer can dispose of with the greatest freedom. Resource distribution must be taken into account since such distributions can affect the aggregate amounts of consumption. The "correct" decision unit must also be considered in an aggregate model.

---

1. In the theoretical framework of Section I this would mean that

$$\frac{\partial W_t^*}{\partial A_t} < 0 \quad \text{implies} \quad \gamma_4 < 0$$

$$\frac{\partial C_t}{\partial A_t} > 0 \quad \text{implies} \quad B_5 > 0$$

Finally, there is the need to make allowance for factors other than resource variables, such as the age distribution of the population.

SECTION III

Statistical Analysis and Empirical Results

Sections I and II presented a theory of consumer behavior and discussed some of the aggregation problems encountered when moving from the consuming unit to an aggregate function. Income, initial net worth, capital gains, income distribution, net worth distribution, capital gains distribution, and age distribution are significant in such a function. Liquid assets are important only when they are used as a proxy for net worth.

It is the purpose of this section to present the results of the regression study which examined these variables, and to discuss the statistical significance of these results with reference to the theory of Sections I and II.

A. Statistical Results

Ordinary least squares regression analysis was used in this study, with observations covering the years 1901 through 1958. Observations longer than a year were weighted in the regression analysis since the theoretical model requires observations of one year. (For a complete discussion of this weighting, see Part A of the Appendix.) Altogether there were 19 observations, one of which covered the period 1940 through 1945. This observation was not included in the regression analysis since the period contained years during which consumption goods were rationed. (For a list of data and their sources, see Part B of the Appendix.) Consequently the regression estimates were based upon 18 observations.

The estimated results of this analysis are presented in Tables 3 through 6 of the Appendix. Tables 3 and 4 are based upon current values per head; Tables 5 and 6 are based upon real values per head. The upper figure in each box of these tables represents the estimated coefficient.[1] The lower figure is the "t" statistic associated with the estimated coefficient. The basic regression format is at the top of the table; the variables used in each specific format (as numbered at the left of the table) are shown by the completed boxes in the table. All four tables have the same 15 formats. Only the constant term and the disposable income variable were included in all runs.

The format derived from the theory presented in Sections I and II includes all variables except liquid assets and is presented in Table 1.[2] (The actual test, however, does not include the distribution of net worth or of capital gains since data were not available on these two variables. See the discussion in Section II.) The results using only 13 observations for this

1. All estimation was done with an ordinary least squares regression program on an IBM 709 computer at the Yale University Computing Center.

2. This format is occasionally called regression format 16.

TABLE 1

Regression Results of the Theoretical Model

$$C_t = B_1 + B_2Y_t + B_3W_{t-1} + B_4G_t + B_5A_t + B_6D_t$$

17 observations

	B_1	B_2	B_3	B_4	B_5	B_6
1	159.97 1.99	.457 7.66	.063 5.19	.035 1.87	-1.458 - .22	-3.158 -2.07
2	149.54 3.22	.476 8.54	.057 5.44	.034 1.64	2.243 .56	-3.162 -3.75
3	143.22 .77	.460 5.40	.030 2.31	-.011 -.51	28.79 1.96	- .395 - .12
4	175.87 1.08	.442 5.77	.026 2.16	-.017 -.82	33.19 2.82	- .772 - .27

13 postwar observations

1	71.97 2.06	.382 4.30	.080 3.57	.031 1.79
2	82.62 2.43	.384 4.64	.078 3.79	.032 1.66
3	113.78 1.49	.519 3.57	.052 2.80	.015 1.02
4	126.96 1.60	.522 3.59	.049 2.73	.014 .88

1. Corresponds to money values as in Table 3.
2. Corresponds to money values weighted by the reciprocal of initial net worth as in Table 4.
3. Corresponds to real values as in Table 5.
4. Corresponds to real values weighted by the reciprocal of initial net worth as in Table 6.

(Upper figure is the estimated coefficient; lower figure is the t statistic.)

format are also presented in the second part of Table 1. (However, the age and income distribution variables were not included in the equation since they changed very little during the period. This omission will be discussed below.) These observations covered only the postwar period, i.e., 1946 through 1958. (The regression formats of Tables 3 through 6 were also estimated with these 13 observations, and the results were statistically the same for the resource variables as those obtained with 18 observations. This point will be discussed below, but the regression results are not listed since they are similar to those of Tables 3 through 6; however, any exceptions are noted below.)

The statistical results on all four tables are basically the same, with the exception of one or two variables. The constant term, when significant, is positive and varies by several hundred dollars. Since the study has focused attention upon a theoretical model along the format of Table 1, it will concentrate upon the results of that table; contrasting references will be made to the other tables. Also the theoretical model focuses upon real variables, and thus Tables 3 and 4 will be used in comparison with Tables 5 and 6 to note evidence of money illusion.

In Table 1, the constant is at a level of approximately $150 per person both in real and money terms, but appears to be significant[1] only in money terms.

The second coefficient estimate is the marginal propensity to consume out of disposable income. It should be noted that if initial net worth is omitted from the regression equation (formats 1, 4, 8, 10, 11, 12, and 13), the estimated coefficient is much higher than if net worth is included. The coefficient estimate is between .7 and .9. Once initial net worth is included in the function, this coefficient drops substantially; in tests of Table 1, it is about .45 (other tests range from .4 to .6), which means that people would consume $.45 out of an additional dollar of disposable income. The theoretical analysis of Sections I and II requires that net worth be included; and,

---

1. The term "significant" is used in the statistical sense with the standard (one-tail) 95% level. If the t value is one which occurs 5% of the time or less, then one rejects the null hypothesis, and the estimated coefficient is denoted significant, i.e., rejects the null hypothesis and accepts the alternate hypothesis that the coefficient is significantly different from zero. If the t value is one which occurs 6% of the time or more, then one cannot reject the null hypothesis that the coefficient equals zero. However, this differs from asserting that it is zero; this means that the data appear to be consistent with the null hypothesis but does not prove it.

Strictly speaking, these tests of significance are not applicable to such a time series study. For a discussion of this problem, see Trygve Haavelmo, "Methods of Measuring the Marginal Propensity to Consume," Chapter IV, Studies in Econometric Method, edited by William C. Hood and Tjalling C. Koopmans (New York: John Wiley & Sons, Inc., 1953), pp. 75–91.

therefore, the estimated coefficient of disposable income is assumed to be in the lower range. Without the net worth variable, the estimated coefficient is comparable to the results of other time series studies.[1] All of the regression formats in Tables 1 and 3 through 6 showed income to be a highly significant variable; the t levels were all in the range of .999 significance. This result has been anticipated in other studies, but the drop in the estimated coefficient when net worth is included is worth noting and will be discussed below.

The next variable is initial net worth. As noted above, when such a variable is included in an equation, the estimated income coefficient falls. The net worth coefficient is always positive and highly significant. Its range is within the .03 to .05 level; however, such a "low" coefficient still implies a high level of consumption since net worth is a large magnitude.[2] A net worth of $1.6 trillion, as in 1958, means an addition to consumption of approximately $60 billion ($B_3 = .04$). This initial net worth or wealth effect has been called the Pigou effect in Section I. (Pigou talked of the size of net monetary wealth changing due to falling price levels, but effectively he was talking of the influence of wealth upon consumption. Patinkin explicitly made allowance for the stock in the consumption function discussed above in Section I and in the Preface.)

Furthermore, it should be noted that this effect is not a "one-shot" affair but will repeat itself period after period. The key variable is the stock. If it remains at a high level, consumption will remain at its comparably high level. If net worth rises and stays at a new level, consumption will rise and stay at a new level. If net worth falls because of price changes, there is not only the change itself to consider (the flow or capital gain) but there is the new lower stock. Its effect continues into the future. A sharp decline in the stock market decreases net worth and is a depressant (upon consumption) which continues not only in the year of decline but into the

---

1. For examples, see Robert Ferber, A Study of Aggregate Consumption Functions (New York: National Bureau of Economic Research, Inc., 1953), Technical Paper No.8.

2. In terms of consumption, the coefficient B_3 is significant in magnitude; however, $B_3 = \theta$ and θ appears to be "low" when considered in relation to the basic behavioral equation of the theoretical model, $S_t = \theta[W_t^* - W_{t-1} - G_t^e]$. An explanation of this "low" valued coefficient is that the gap must be large due to a high level of target wealth. This target wealth is a multiple of current income (roughly 9 times—see Part C of the Appendix, Table 10, of John J. Arena, The Wealth Effect and Consumption: A Statistical Inquiry (New Haven: Yale University, 1963). Slight changes in income mean large changes in the desired level of wealth. Saving, which is only a part of income, would appear small in relation to target wealth; consequently θ would be small. (Part D of this section makes some estimates of desired wealth. Income, and especially saving, is small in relation to these estimates.)

future, until the stock is rebuilt through saving or capital gains. Nevertheless, the empirical evidence clearly lends support to the theoretical Pigou or wealth effect. Initial net worth is significant in its effect upon consumption.[1]

The next variable to consider is the flow of wealth or, specifically, capital gains. In the tables, the capital gains coefficient seems to vary between positive and negative in sign as well as in significance. In current values (Tables 3 and 4), the coefficient is negative when initial net worth is excluded as an exogenous variable but positive when net worth is included. (This seems to imply a negative correlation between capital gains and wealth.) However, on theoretical a priori grounds the results of the regression equations which include net worth should be emphasized (the focus of attention should be upon the results of tests that include net worth as a variable). Such tests imply that the null hypothesis that capital gains have no effect upon consumption ($B_4 = 0$) should be rejected and the alternate hypothesis that capital gains lead to more consumption ($B_4 > 0$) should be accepted.

The implications of Tables 5 and 6 are somewhat similar. If net worth is omitted, the capital gains coefficient drops, usually to negative values (which are statistically significant). If net worth is included, however, the capital gains coefficient increases and is either slightly negative or positive but not significant. The capital gains coefficient cannot be claimed to differ from zero. (It should be noted the statistical results concern real capital gains which have not been computed by the simple deflation of money capital gains. For a discussion of this real capital gains variable, see Part B of the Appendix. A few test regressions using such simply deflated "real" capital gains were performed, but are not shown here. The results were essentially those of Tables 3 and 4 which examined money values. All estimated coefficients were positive and occasionally significant when initial net worth was included; all estimated coefficients were negative and significant when initial net worth was omitted.)

In Table 1, which is the theoretically preferred model, the capital gains coefficient is again positive in money terms and negative in real terms, but not significant. (Seventeen observations, covering 1913 through 1939 and

---

1. One further step was undertaken to test the significance of the net worth coefficient. Since net worth could be only a proxy for some unnamed growth item, a regression analysis was performed with a consumption function which incorporated time, T_t, as an explicit proxy for the growth item. The regression test was of the following two formats (with 18 observations):

$$C_t = B_1 + B_2Y_t + B_3W_{t-1} + B_4G_t + B_5T_t$$

$$C_t = B_1 + B_2Y_t + B_3W_{t-1} + B_4G_t + B_5A_t + B_6T_t$$

In all tests, the time variable (T_t) coefficient was significant, but the coefficient of net worth was also significant with t values of approximately 3.

1946 through 1958, were used.) However, when only the postwar period is examined, 1946–1958, the estimated coefficients are all positive (but not significant). All formats in Tables 3 through 6 were examined for the postwar period (which covers only a quarter, approximately, of the time span of the 18 observations). In the great majority of such runs, with or without initial net worth, and whether in money or real terms, the estimated coefficients of capital gains were positive and often significant.

The reason for examining only the postwar data was to test the weighting technique for the longer periods. These postwar data were the only observations on an annual basis. All other prior data covered periods longer than a year and consequently were weighted in the regression tests to make them comparable to the annual data. The results were then compared to those obtained by using all the data (except the observation for the war years.) The major variables of income and net worth stayed almost the same in size, significance, and sign. The coefficient estimates of capital gains did very slightly as mentioned above. All deviations are discussed in this chapter when the variable is considered.

Nevertheless, the statistical results are not clear cut, but within the theoretical framework of the regression tests that include net worth, it can be said either that capital gains have a positive effect upon consumption ($B_4 > 0$), or that from these data one cannot tell whether capital gains affects consumption. In all the tables, the capital gains coefficient was never significantly negative if initial net worth was in the equation. [1]

Furthermore, a low positive capital gain coefficient could be the result of unanticipated capital gains. The coefficient of capital gains is B_4 which

---

1. One further step was taken to test the significance of a positive B_4. If capital gains are capitalized into a stock and consumed as if they were the same as the stock of wealth, this would imply that both initial net worth and capital gains would have the same coefficient, i.e., $B_3 = B_4$. To test this hypothesis that $B_4 - B_3 = 0$ was postulated, and the basic regression format,

$$C_t = B_1 + B_2Y_t + B_3W_{t-1} + B_4G_t ,$$

was changed by adding and subtracting B_3G_t, so that

$$C_t = B_1 + B_2Y_t + B_3(W_{t-1} + G_t) + (B_4 - B_3)G_t$$

was obtained. ($B_4 - B_3$ was set equal to B_8). The regression coefficients were computed. The estimated coefficient B_8 was slightly negative but never significantly different from zero, that is, it cannot be asserted from this evidence that B_3 and B_4 differ significantly from one another. B_3 is a positive and significant coefficient; B_4 may also be positive and significant. There is not sufficient information to determine the validity of the latter statement. Whether $B_4 = 0$ or whether $B_4 = B_3$ cannot be distinguished from these data; the test is of insufficient power to make this distinction.

equals $\theta(1 - \gamma_3)\phi_2$. ($G_t^e = \phi_1 + \phi_2 G_t$). If expected capital gains are small relative to actual gains, i.e., ϕ_2 is close to zero, then the coefficient on capital gains, B_4, will tend toward zero. Consequently, the existence of a low coefficient on capital gains could be attributable to unanticipated gains. (A similar statement could be made regarding a "low" coefficient of income and the existence of unanticipated income—λ_2 is close to zero.) Such gains, however, do become part of the initial stock of wealth in the next period.

A negative coefficient on capital gains could imply that γ_3 is greater than one ($W_t^* = \gamma_1 + \gamma_2 Y_t^e + \gamma_3 G_t^e + \gamma_4 A_t$), which would mean that although the consumer possesses more resources with the receipt of a capital gain, the gain has increased the desired wealth to such a degree that the gap ($W_t^* - W_{t-1} - G_t^e$) has <u>increased</u>, causing the consumer to save more, i.e., to consume less. This possibility is consistent with the general model but contradicts one of the additional assumptions placed upon the model that B_4 should be positive.

In any case, the capital gains is a flow variable. It becomes part of the stock of initial wealth for the next period. Initial net worth is a stock that is positively and significantly related to consumption. Its effect is felt period after period. The capital gains is the flow or "one-shot" factor; it is the flow that Tobin discusses in the passage quoted in the Preface. Such flows appear to be capitalized into net worth as both the Friedman[1] and Modigliani and Brumberg[2] models would imply.

The next variable considered is initial holdings of liquid assets. If initial net worth is excluded from the equation, liquid assets appear to encourage consumption since the estimated coefficient is both positive and significant for all tables. However, when initial net worth is included, the estimated coefficient usually becomes negative and is always insignificant, that is, liquid assets do not appear to influence consumption. One very plausible interpretation of these results which is consistent with the theory of Section I is that liquid assets are significant only as a proxy for net worth. When net worth is included, the sign and significance of liquid assets fall.

The results derived with the postwar data are slightly different. Again if net worth is excluded, liquid assets is usually a significant variable (whose coefficient is greater in magnitude, averaging around .35) for both money and real values. When net worth is included, the size of the coefficient falls (to the range .02 to .06) but remains positive and is significant (as in tests 14 and 15). One explanation of these results is that there exists a "ratchet effect" in consumption behavior as set forth by Duesenberry.[3] In a short

---

1. Friedman, <u>op. cit.</u>

2. See discussion of Section I.

3. James S. Duesenberry, <u>Income, Saving and the Theory of Consumer Behavior</u> (Cambridge, Mass.: Harvard University Press, 1945),

period—e.g., 1946 through 1958 with its slight recessions in 1949, 1954, and 1958—people may not wish to lower substantially their standard of living when income falls. Consequently, to maintain a high consumption level, they consume part of their liquid assets. (There is little or no cost to the individual of turning these assets to cash.) Liquid assets in and of themselves do not encourage more consumption; they are not the causal factor. They merely provide the necessary liquid resources to maintain consumption. Theoretically, only the three resources of income, capital gains, and total net worth would be causal factors. However, due to the ratchet effect in this short period, liquid assets are correlated with consumption. But over the longer period—1913 to 1958—such assets appear to be insignificant in regard to consumption. Consequently these statistical results are not inconsistent with the theoretical implications of Section I. (See Part C of the present section for a further note on this matter.)

The last two variables in the regression study are age distribution and income distribution. Over a long time period during which these variables can change substantially, they can have an impact upon aggregate consumption.

The age distribution is the per cent of total population over 65 or roughly the per cent of people in the retirement span of the Modigliani and Brumberg model.[1] The statistical results are all positive, which confirms the theoretical implications of the basic model. In the money values of Tables 3 and 4, this variable is not always significant; however, in real values of Tables 5 and 6 it is always significant. Aggregate consumption per head rises in the range of $29 to $42 with each one point increase in the percentage of total population in the 65 or older bracket. The same results are implied in Table 1 for real values, but the coefficient for money values does not differ significantly from zero.

Data from the postwar period tend to give peculiar, though what appear to be statistically significant, results. These results imply that consumption falls (by around $125 per head) as a larger proportion of the population enters the 65 or older age bracket. However, the results of the constant term, the age distribution variable, and the income distribution variable in all tests of 13 observations should be interpreted with caution because of the fact that there has been little or no change in the latter two variables over this short time period and the constant term does not change at all (equals 1) as a regression input for this period. Consequently, in the computations of the coefficients, there is great difficulty in trying to distinguish between the three variables since there is not sufficient information. It is for this reason that an abbreviated regression format, which omits the age and income

1. See Section I. While the authors had no set age for retirement, 65 was chosen as this age since most pension plans, public and private, use this figure. The greater the number of people in the retirement or dissaving stage, the higher should be aggregate consumption.

distribution variables, is presented in the second half of Table 1. (However, over the longer 59-year span, these variables have changed, and these results should be emphasized. See Table 2 of Appendix.)

In all the results using only 13 observations where age distribution was a variable (formats 3, 7, 15, and 16), the constant term increased immensely, and the estimated coefficient of age became significantly negative. The results for these variables based upon only the postwar period should be disregarded, and the observations of the longer time span (which includes the observations of the postwar period) should be accepted.

The last variable to be examined is income distribution, which is the per cent of total disposable income received by the upper 5% of such income recipients. This is a somewhat crude measure of income inequality but should be acceptable if the marginal propensity to consume of this higher income bracket is lower than that of other brackets; therefore, the larger the per cent of income received by this group, the lower should be aggregate consumption. In Tables 3 through 6, the estimated results appear to be negative and significant approximately half of the time. The real values of Tables 5 and 6 are less often significant. Nevertheless, the negative sign implies that aggregate consumption does fall (by $2 or $3 per head) as the per cent of total income of the upper 5% rises. Table 1 implies that the effect is negligible in real terms, or rather, these statistical data do not contradict the hypothesis that this variable has no effect on aggregate consumption ($B_7 = 0$). Again the statistical results from only the postwar period should be viewed with caution. Furthermore, the data for this variable in the long period are not consistent with one another so that all the results for this variable must be interpreted with a great deal of skepticism. (See Part B of the Appendix for a discussion of this variable.)

B. Summary and Conclusions

What then are the conclusions from this analysis? First of all, income and initial net worth have consistently appeared to be important factors influencing consumption; they are highly significant in both real and money values, confirming the theories of the Pigou wealth effect. Second, consumption out of capital gains is probably slightly positive and at least significantly so in future periods when the current period's gains are then part of net worth, thus tending to confirm parts of the Friedman and Modigliani and Brumberg theories. Third, consumption out of liquid assets tends to be an insignificant factor upon consumption in the long run but in the short run may enable consumers to temporarily adhere to old higher consumption patterns. Fourth, over long time periods, as the population ages, aggregate consumption rises. Finally, consumption appears to decline somewhat as the higher income brackets receive more of total income.

As for the basic hypothesis of Section I, it is confirmed by this evidence. The Pigou or wealth effect, which is the stock or net worth effect, is highly significant both in statistical terms and quantitative terms. It is

not a "one-shot" affair. The flow or capital gains component, that could be interpreted as part of the Pigou effect, also appears to influence consumption, but the statistical results are not as clear cut as they were with the stock. The other variables are also important, but again the evidence is not absolutely clear.

C. A Few Supplementary Remarks

There are certain theoretical implications of the statistical results which should be noted. First of all, a simple reconciliation can be obtained between the apparently different results of time-series and cross-section studies. The two basic differences are: first, cross-section results imply that there is a larger intercept or constant term than that of time-series data; second, the marginal propensity to consume out of income is lower in cross-section studies than in time-series studies (particularly at the upper income levels.) In this study such results need not contradict one another. The time-series results above imply a consumption function as follows:

$$C = K + .5Y + .05W$$

(K here is the constant term; the numbers were chosen as a rough average from the tables above.)

The cross-section results, which omit wealth (net worth) as a variable imply a function as follows: $C = K' + .5Y$. In the short run, W is relatively fixed ($\overline{W}$); therefore, $K' = K + .05\overline{W}$. The short-run cross-section results must have a larger intercept due to the omission of the initial net worth variable. The estimated coefficient out of income, however, is comparable for both.

Conversely, if net worth is omitted from time-series, during which wealth does vary, then the income coefficient, rather than the constant term, rises substantially as seen in the empirical work above. Other studies have only the higher estimate of the income coefficient. The coefficient appears to "capture" some of this wealth influence, rather than the constant's absorbing it as in cross-section studies. The difference should be resolved by the inclusion of initial net worth as an additional exogenous variable for both studies.

A final factor worth noting is the built-in stability or ratchet effect of consumption due to the net worth component. Consumption could exceed income in short periods. If the consumption function were

$$C_t = .5Y_t + .05W_{t-1}$$

then consumption could exceed income if income fell to the extent that initial net worth was sufficiently large when compared to income (10 times as large or more). A sudden drop in income as in a recession would be cushioned by wealth. Wealth will change but will do so slowly unless there are large capital losses. Liquid assets could provide the short-run source of funds to maintain the relatively high level (to income) of consumption.

D. Estimated Consumption and Wealth in the Postwar Period

For a final check of the theory and statistics above, estimated consumption and wealth figures were computed for the postwar period. Using the coefficients of Table 1, the estimated consumption figures for the postwar period were computed.[1] The money and real estimated values, derived from the coefficients of data which are weighted by the reciprocal of initial net worth and which cover the period 1913–1939 and 1946–1958, are plotted in Figures 9 and 10 along with the actual values of this postwar period, 1946–1958. The coefficient estimates of the longer period were used for estimation in the shorter period. (This particular series is plotted because it is derived from the theoretically preferred set of coefficients. For a discussion of this preference, see Part A of the Appendix concerning the weighting of data.

The estimated series of consumption in money values is quite close to the actual series as in Figure 9. The residuals are relatively small, and are presented in the lower half of the figure. The estimates in real terms differed more from the actual series as can be seen in Figure 10; the residuals in the second half of Figure 10 are larger on the average than those in Figure 9.

The basic pattern is roughly the same for the other series. The estimates in money terms tend to have smaller residuals than those in real terms.

Figures 11 and 12 present an analogous series for wealth in the postwar period both in money and real terms. Again the money values have smaller residuals,[2] and again both series were derived from the coefficient estimates of data of the longer period (1913–1939 and 1946–1958) weighted by the reciprocal of initial net worth.

The statistical problem of identification arose, however, in the model so that it was not possible to obtain estimates of the parameters of the wealth equation. However, by making the additional assumption that the expected value of capital gains is zero, i.e., $\phi_1 = 0$ and $\phi_2 = 0$, the wealth coefficients can be derived. This additional constraint was placed upon the original model, and a new consumption function was derived which does not contain capital gains as an exogenous variable, i.e., $C_t = B_1 + B_2Y_t + B_3W_{t-1} + B_4A_t + B_5D_t$. (All capital gains are now assumed to be unexpected and affect consumption only insofar as they become a part of the stock of wealth in future periods.) The coefficients of this consumption function were estimated, and from these coefficients, the coefficients of the wealth equation

1. All computations mentioned in this part are presented in John J. Arena, op. cit., Appendix.

2. The term residual does not mean the difference between the actual and estimated value of a variable as it did for consumption. Here the term simply denotes the difference between actual ending wealth and estimated desired ending wealth.

were derived. These coefficients were used to compute the desired wealth figures for the postwar period. Again, only those results derived from coefficients of data of the longer period, weighted by the reciprocal of initial net worth, have been plotted; they are in Figures 11 and 12.

The desired ending wealth always exceeds the actual ending wealth, but the series are roughly parallel to one another. Only in the second and last years (1947 and 1958) does the estimated value fall while the actual value rises. This is due to the fact that actual income fell, causing desired wealth to fall, while actual wealth rose because of a large capital gain for the period. (Again it should be noted that such gains are assumed to be unanticipated for these computations.)

The second half of Figure 11 plots the excess of the estimated desired wealth over the actual wealth as a percentage of actual wealth. This excess appears to average approximately 30% in this period. In Figure 12 (real values) it is approximately 45%. Target wealth grows in close relation to income; desired wealth is a multiple of current income. Saving in this postwar period is a small percent (roughly 7% in money terms and 5% in real terms) of the gap between actual and desired ending wealth. Even if it were possible to save all income (and to remain at a high level of output), the gap could not be filled since this gap is greater than income. Such a large gap is one explanation of the "low" estimated value of θ in the saving equation of the model; target wealth appears to be at a relatively high level.

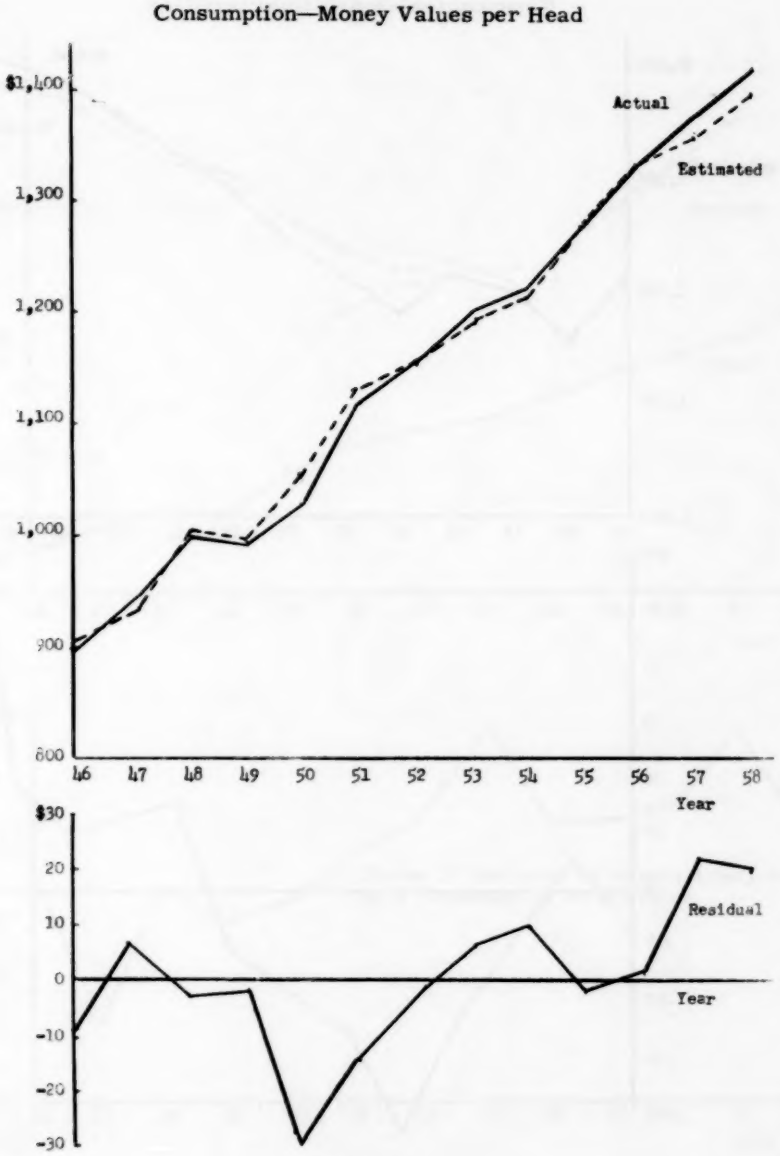

Consumption—Money Values per Head
$1,400
1,300
1,200
1,100
1,000
900
800
Actual
Estimated
46
47
48
49
50
51
52
53
54
55
56
57
58
Year
$30
20
10
0
-10
-20
-30
Residual
Year

Figure 9

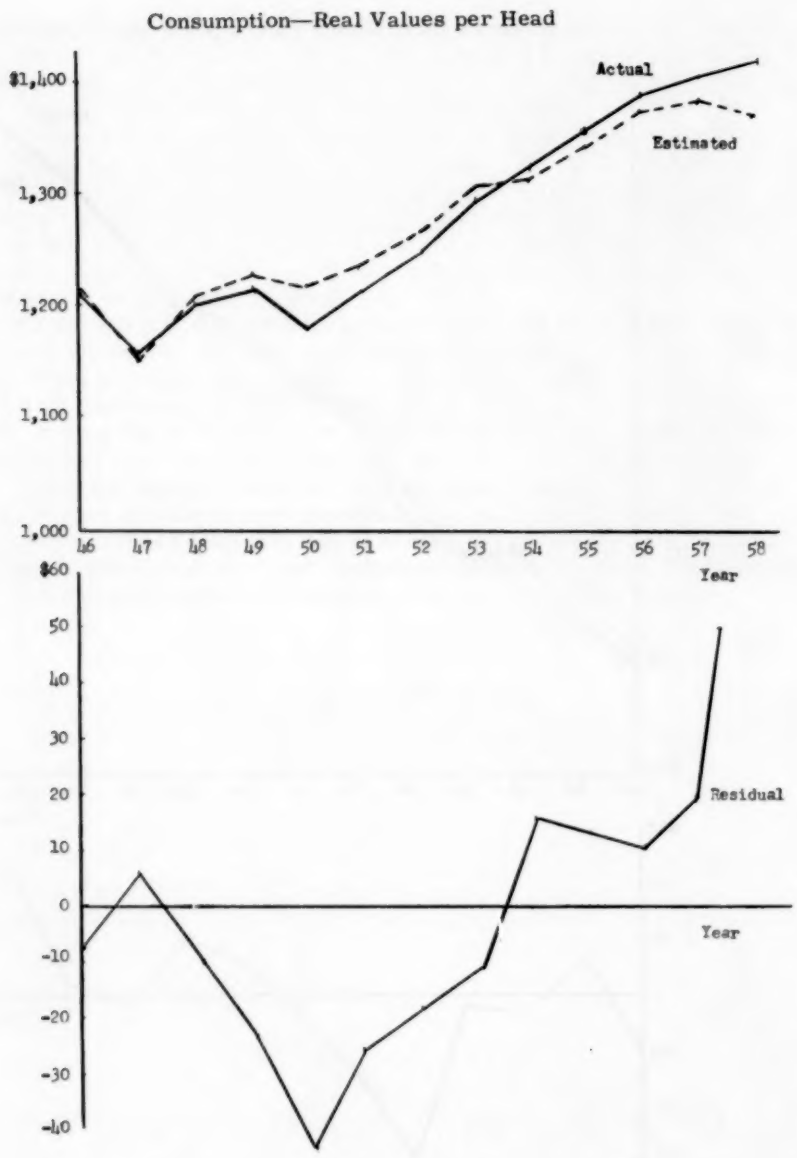

Consumption—Real Values per Head
$1,400
1,300
1,200
1,100
1,000
Actual
Estimated
46
47
48
49
50
51
52
53
54
55
56
57
58
Year
$60
50
40
30
20
10
0
-10
-20
-30
-40
Residual
Year

Figure 10

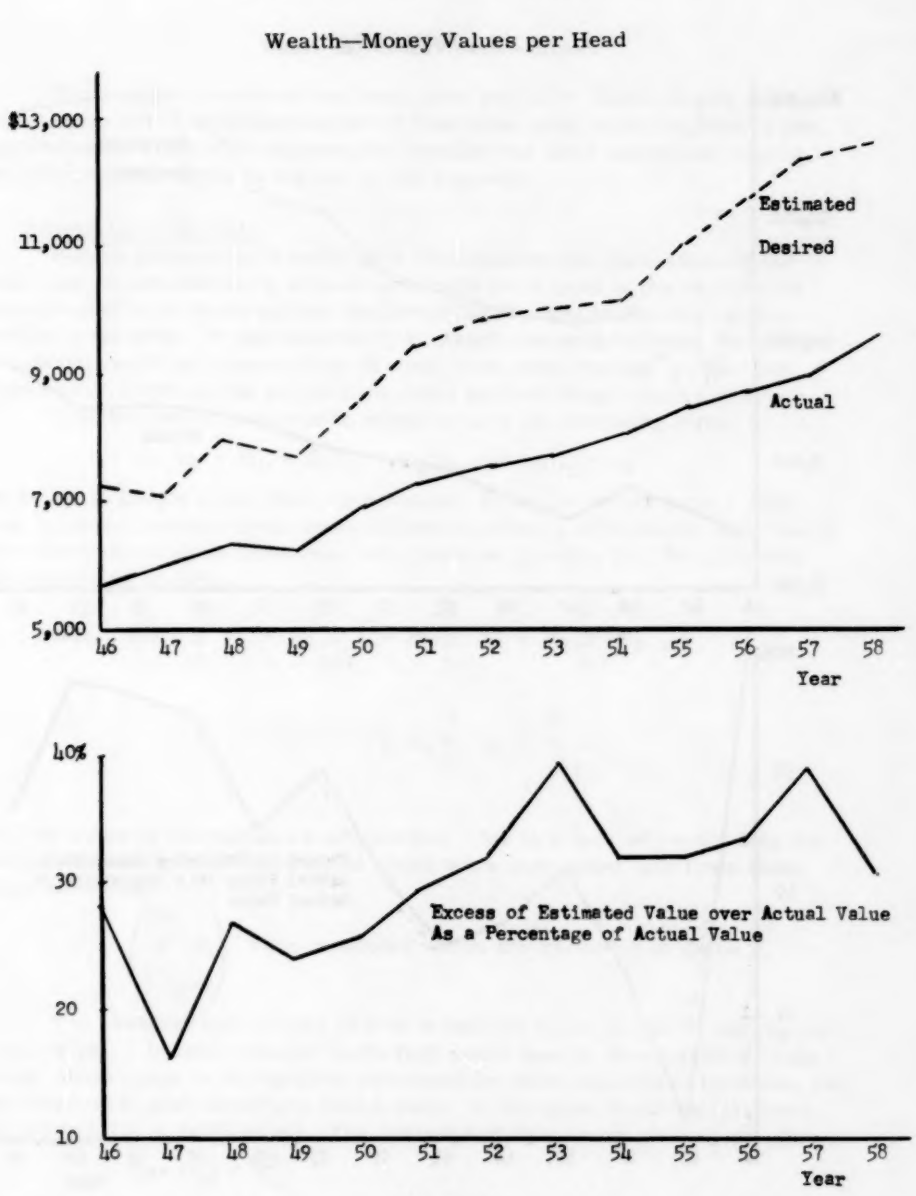

Wealth—Money Values per Head
$13,000
11,000
9,000
7,000
5,000
Estimated
Desired
Actual
46 47 48 49 50 51 52 53 54 55 56 57 58
Year
40%
30
20
10
Excess of Estimated Value over Actual Value
As a Percentage of Actual Value
46 47 48 49 50 51 52 53 54 55 56 57 58
Year

Figure 11

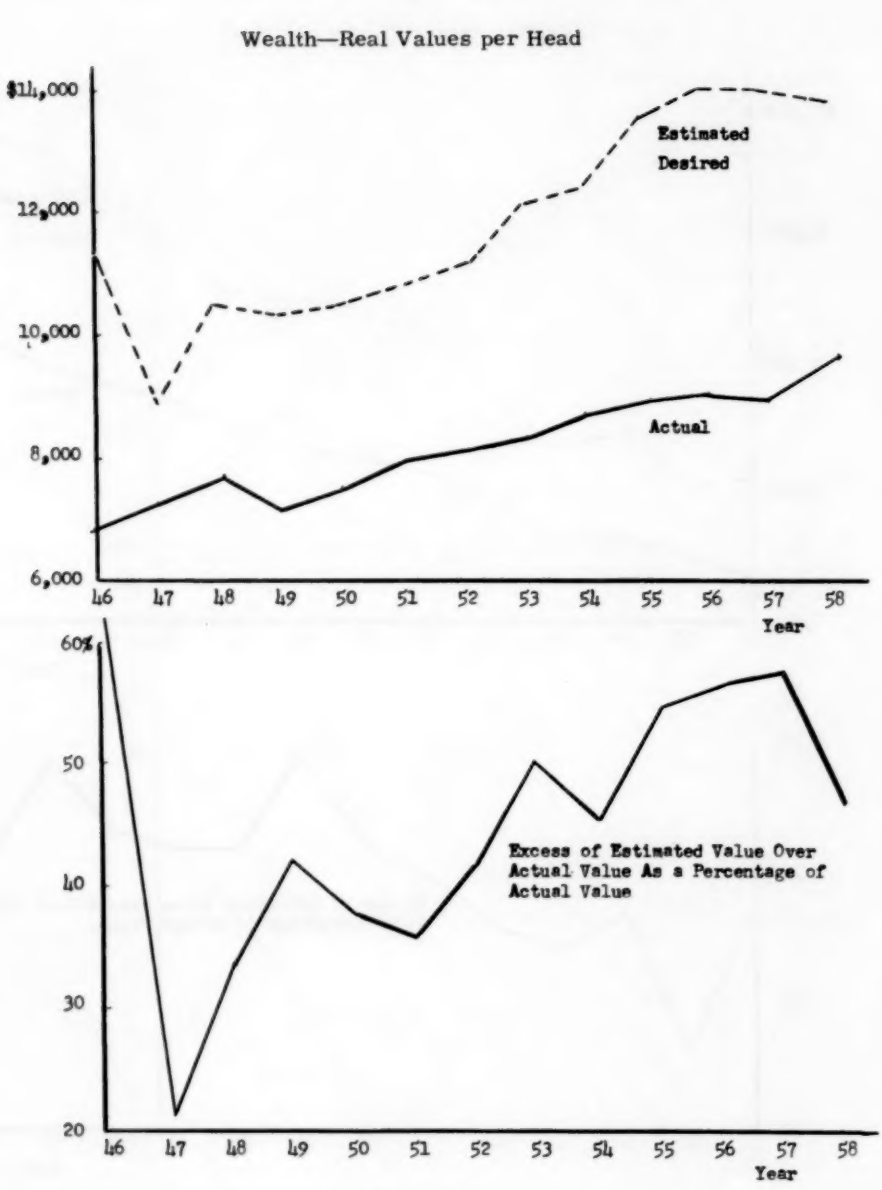

Wealth—Real Values per Head
$14,000
12,000
10,000
8,000
6,000
Estimated Desired
Actual
46 47 48 49 50 51 52 53 54 55 56 57 58
Year
60%
50
40
30
20
Excess of Estimated Value Over Actual Value As a Percentage of Actual Value
46 47 48 49 50 51 52 53 54 55 56 57 58
Year

Figure 12

APPENDIX

The Appendix presents the basic data and their sources used in this study, along with a brief discussion of how these data were weighted in the regression formats. The regression formats and their statistical results are presented in tables at the end of the appendix.

A. Weighting of the Data

Before proceeding directly to a presentation and discussion of the basic data, a consideration of how these data were used in the regression analysis should be made so that the format of the data in Table 2 can be readily understood. It was necessary to weight the data because the theoretical model requires observations of equal time span, namely a year; but some observations in the actual data cover periods longer than a year.

The theoretical regression equation is of the following form:

$$(1)\quad C_t = B_1 + B_2Y_t + B_3W_{t-1} + B_4G_t + u_t$$

where t represents a one-year observation. (u_t is the error term.) The data, however, contain some observations consisting of N years. For this N year observation to be consistent with those of Equation (1), the following information is needed.

$$(2)\quad \sum_{t=1}^{N} C_t = B_1 \sum_{t=1}^{N} 1 + B_2 \sum_{t=1}^{N} Y_t + B_3 \sum_{t=1}^{N} W_{t-1} + B_4 \sum_{t=1}^{N} G_t + \sum_{t=1}^{N} u_t$$

i.e., the sums of the variables are needed. Due to a lack of yearly data for certain variables, it is these sums which were calculated, and from these sums the means

$$\left(\frac{\sum_{i=1}^{N} x_i}{N} \right)$$

were computed which are recorded in Table 2.

The problem then arises of how to include these means in the regression analysis. In their present form they would tend to be equal only to an annual observation in an equation estimated by least-squares calculation, yet they represent periods longer than a year. In the ideal Equation (1) above, the error term u_t is included. The variance of this error term is $\sigma^2_{u_t}$, i.e.,

$$\text{Var}\,(u_t) = \sigma^2_{u_t}$$

In computing the means from Equation (2) we now have $\frac{\sum_{t=1}^{N} u_t}{N}$ and not merely u_t. The variance of $\frac{\sum_{t=1}^{N} u_t}{N}$ is $\frac{\sigma^2_{u_t}}{N}$, i.e.,

$$\text{Var}\left(\frac{\sum_{t=1}^{N} u_t}{N}\right) = \frac{\sigma^2_{u_t}}{N}$$

The variance of this error term is smaller (by the reciprocal of the number of years in the observation) than the variance of a one-year observation. In order to make the variances of the observations comparable to one another, each observation should be weighted by the square root of the number of years in the observation, i.e., the observation should be multiplied by $\sqrt{N}$ so that the equation being used is:

$$(3)\quad \frac{\sum_{t=1}^{N} C_t}{N}\cdot\sqrt{N} = B_1\frac{\sum_{t=1}^{N} 1}{N}\cdot\sqrt{N} + B_2\frac{\sum_{t=1}^{N} Y_t}{N}\cdot\sqrt{N} + B_3\frac{\sum_{t=1}^{N} W_{t-1}}{N}\cdot\sqrt{N} + B_4\frac{\sum_{t=1}^{N} G_t}{N}\cdot\sqrt{N} + \frac{\sum_{t=1}^{N} u_t}{N}\cdot\sqrt{N}$$

The variance of $\frac{\sum_{t=1}^{N} u_t}{N}\cdot\sqrt{N}$ is $\sigma^2_{u_t}$, i.e.,

$$\text{Var}\left(\frac{\sum_{t=1}^{N} u_t}{N}\right)\cdot\sqrt{N} = \text{Var}\left(\sum_{t=1}^{N} u_t \Big/ \sqrt{N}\right) = \frac{N\sigma^2_{u_t}}{N} = \sigma^2_{u_t}$$

Thus all observations were actually weighted by the square root of the number of years in the observation. A single-year observation is weighted by one. This weighting is employed in all the regression analysis to adjust for observations of different time span.

One further difficulty encountered with this statistical problem of efficiency arose due to the long time span covered by the observations. Over time there is a change in the magnitude of the variables and a consequent change in the size of the variance, i.e., the problem of heteroskedasticity. (For a brief, simple explanation of this problem, see Stefan Valavanis, Econometrics, McGraw-Hill Book Company, Inc., 1959, pp. 48–49.)

One way to avert this difficulty is to investigate the ratios of the variables to one another rather than the absolute values of the variables. Again the observations are being weighted. Most studies examine the

C_t / Y_t (or S_t / Y_t) ratio, i.e., they weight the observations by the reciprocal of income. However, in the analysis of Section III all variables of a particular observation were weighted instead by the reciprocal of initial net worth, i.e.,

$$(4)\quad \frac{C_t}{W_{t-1}} = \frac{B_1}{W_{t-1}} + \frac{B_2 Y_t}{W_{t-1}} + \frac{B_3 W_{t-1}}{W_{t-1}} + \frac{B_4 G_t}{W_{t-1}}$$

became a basic regression format.

The results of this weighting were discussed in Section III above and are presented in Part C of the Appendix. In each of these tests, all observations were again weighted by the reciprocal of the square root of the number of years.

B. Data

Table 2 presents the basic data used in this study. Additional variables were considered, such as different definitions of saving and consumption, and different deflators, such as the GNP price index or the number of households; but the statistical results obtained from this extra information are essentially the same as those derived from the data in Table 2. Therefore, only the basic data, from which the results of Section III were obtained, are presented in the table.

As can be seen in Table 2, some observations covered periods longer than a year and were the result of insufficient data concerning initial net worth. The following discussion is a detailed breakdown of each column in Table 2.

Column 1: Saving

Section I focused upon the behavior of the individual consuming unit whose aggregate counterpart is the household sector of the private sector in the economy. This saving column, therefore, lists the personal saving of the household sector which consists of three subsectors: nonagricultural individuals, agriculture, and unincorporated business. Corporate business and the government are excluded from the definition of the household. (While there is some business activity in the agricultural and unincorporated sectors, the distinction between personal and business saving is not discernible since the individual has great control over these amounts, and consequently it is empirically impossible to clearly separate the saving; thus, for pragmatic reasons, all such saving was included in the definition of household saving for this study.)

The composition of this saving is not the same as that of the Department of Commerce, but is rather the Social Accounting Concept of saving used by Raymond Goldsmith in his Study of Saving in the United States[1] (henceforth referred to as Saving Study). The two basic differences are that

1. Raymond W. Goldsmith, A Study of Saving in the United States (Princeton, New Jersey: Princeton University Press, 1956), 3 Volumes.

consumer durables are considered a part of saving in the latter definition, and all depreciation is on a replacement-cost basis rather than upon an original-cost basis. This latter difference keeps costs on a current basis, and they are, therefore, more or less comparable to one another. For a detailed discussion and breakdown of the saving definition, see Goldsmith, Vol. I, p. 31.

The figures through 1945 were obtained from Table T-1, Vol. I, p. 345. The observations greater than a year are the mean value of saving for the entire period. The observations 1946 through 1958 were obtained by using the Department of Commerce Personal Saving figures[1] and adding to these the purchase of consumer durables less the replacement-cost depreciation on these durables. The data on the purchase of consumer durables and on the replacement-cost depreciation of such durables were obtained from Professor Goldsmith, who will publish them in a forthcoming study.

One factor should be noted. The 1946 through 1949 figures derived by the above method are not the same as those in the Saving Study (though they are roughly similar). While it is true that the earlier observations may not be strictly comparable with later ones, the figures are close to one another. (The annual observations, i.e., only those observations derived by the method above, were used in one regression series, the results of which were compared to those derived from the inclusion of more observations. The results were statistically the same regarding the significance and approximate size of the resource coefficients; see Section III above.)

Column 2: Consumption

This column represents the consumption of the household sector of the economy and was derived by subtracting the personal saving of the household (Column 1) from personal income (Column 3) of the household.

Column 1 + Column 2 = Column 3

S + C = Y

Column 3: Income

This column is the disposable income of the household sector. It differs slightly from that of the Department of Commerce in that it is net of replacement-cost depreciation rather than original-cost depreciation. Observations through 1945 were obtained from the Goldsmith Saving Study, Volume III, Table N-1, Column 9, p. 427. Observations for 1946 through 1958 were computed in the same way as those of the earlier years. From GNP, replacement-cost depreciation was deducted rather than original-cost depreciation. The reconciling items from Net National Product to Disposable

1. Department of Commerce, U.S. Income and Output, 1958 Supplement (Washington, D.C. : U.S. Government Printing Office), Table V-1, p. 188. Department of Commerce, Survey of Current Business (July 1961), p. 8.

Income were the same as those used by the Department of Commerce. The Department of Commerce data were obtained from the U.S. Income and Output, 1958 Supplement, op. cit., and from the 1962 Economic Report of the President (U.S. Government Printing Office, 1962). The replacement-cost depreciation figures were again obtained from Professor Goldsmith.

The computation of the annual data is consistent with the computation of the saving data. No rental income, however, was imputed to consumer durables as is discussed above in Section II.

Column 4: Net Worth

This column represents the initial net worth of the household sector at the beginning of the period. Observations through 1945 were obtained from the national balance sheets presented in Goldsmith's Saving Study, Volume III, Tables W-9 through W-15. The annual observations, 1946 through 1958, were obtained from similar tables prepared by Raymond Goldsmith and Robert Lipsey (to be presented in a forthcoming book, Studies in the National Balance Sheet).

The net worth figures were obtained from these balance sheets, the difference between assets and liabilities being computed at current market prices. These balance sheets were estimated at the end of the calendar year, and the net worth figures from these balance sheets were used as the initial net worth for the next year. The household's net worth was composed of the net worths of nonfarm households, of nonfarm, noncorporate business, and of agriculture. The balance sheets contained detailed breakdowns of assets and liabilities including such major items as residential structures, land, demand deposits, common and preferred stock, consumer and mortgage debt, etc.

One additional factor should be noted. The theoretical model uses annual observations; however, balance sheets were available only for select years before the Second World War, specifically at cycle turning points. Balance sheets were available for end-of-the-year values immediately at the beginning of each long-period observation. Thus the longer period observations varied in time span with the availability of the balance sheets. Weighting of these longer observations in the regression has been discussed above.

One further note should be made. If an observation covers N years, then it is necessary to have not the initial net worth of the whole N year period, but rather N initial net worths for each of the N years, i.e.,

$$C_t = B_1 + B_2 Y_t + B_3 W_{t-1} + B_4 G_t + u_t$$

is the basic regression format for any observation year, t; for a longer period, however, the sums should be obtained:

$$\sum_{t=1}^{N} C_t = B_1 \sum_{t=1}^{N} 1 + B_2 \sum_{t=1}^{N} Y_t + B_3 \sum_{t=1}^{N} W_{t-1} + B_4 \sum_{t=1}^{N} G_t + \sum_{t=1}^{N} u_t$$

TABLE 2

Basic Gross Data

	Saving	Consumption	Income	Net Worth	Capital Gains
1901–1912	$ 2550	$ 20115	$ 22665	$ 117072	$ 4100
1913–1922	6214	43719	49932	227004	9966
1923–1929	9559	64000	73559	387945	12841
1930–1933	251	56963	57214	421629	-38301
1934–1939	4093	59275	63368	360913	8440
1940–1945	27938	89696	117634	528460	25814
1946	19854	127235	147089	728310	54296
1947	13311	135695	149006	802460	74189
1948	19504	145757	165261	889960	31426
1949	17187	147297	164484	940990	- 5087
1950	25442	154992	180434	953090	85028
1951	25601	170801	196402	1063560	62079
1952	24771	181179	205950	1151240	20929
1953	27547	191393	218940	1196940	723
1954	25350	197958	223308	1225210	67730
1955	29411	209268	238679	1318290	79329
1956	31049	222854	253903	1427030	56381
1957	30330	236464	266794	1514460	10910
1958	27269	246247	273516	1555700	126324

Dollar figures are in millions — 000, 000.

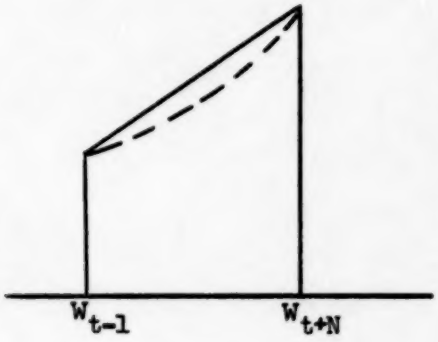
TABLE 2 (cont.)

Real Capital Gains	Liquid Assets	Age Distribution	Income Distribution	CPI Index	Population*
$- 2886	8849	% 4.2	not available	31.67	86384
9064	25270	4.6	% 30.09	49.97	103495
47474	47044	5.1	33.10	59.95	117229
-75794	49202	5.5	32.05	48.88	124384
- 5353	52402	6.4	29.26	48.15	128535
- 2649	96829	7.2	19.78	57.52	135908
-23062	165280	7.7	18.77	74.24	141389
70059	177940	7.8	18.71	81.41	144126
60257	183000	7.9	18.59	83.01	146631
-82603	186620	8.0	18.87	81.41	149188
40555	189150	8.1	19.20	87.09	151683
57137	192600	8.2	18.40	91.36	154360
11255	199040	8.3	18.20	92.07	157028
9209	208780	8.4	17.70	92.89	159636
69676	218630	8.4	18.30	92.30	162417
39808	229870	8.5	18.10	92.54	165270
8505	237100	8.6	18.00	95.38	168174
-12296	248500	8.6	18.20	98.52	171229
105415	260050	8.6	18.20	100.00	174054

* Figures are in thousands — 000.

A longer observation calls for ΣW_{t-1} and not merely W_{t-1}.

This sum of net worth was approximated by the following method, and it is from this sum that the mean was computed which is shown in Table 2 for the longer observations. For each period, both the initial and ending net worths were available from the balance sheets, as in the following diagram, where W_{t-1} is the initial net worth of the first year and W_{t+N} is the ending net worth



of the Nth year. The missing initial net worths for the N years (the first one being W_{t-1} itself) were obtained by interpolation. The straight solid line connecting the two end points would indicate an equal absolute or arithmetic growth over the period; the curved dashed line represents an equal growth rate over the period. The latter method of interpolation was chosen, the rates being selected from compound interest tables. Since the available sheets represented cycle turning points, the use of an even growth rate was thought to be the more appropriate method of estimating the changing stock. The figures in Table 2 are these calculated sums of W_{t-1} divided by the number of years for the period in question.

Column 5: Capital Gains

This column is the money capital gains for the household sector, and was computed by calculating the change in net worth from the beginning of the period to the end of the period, and by subtracting from this difference the money saving of the period. Net worth can change only by saving or by price-level movements. Since all the values of the net worth figures are at current market value, price-level changes are included (e.g., an appreciation in the value of stocks) and can be derived in the aggregate as outlined above, i.e.,

$$W_t \equiv W_{t-1} + S_t + G_t \qquad \text{or}$$

$$W_t - W_{t-1} = \Delta W = S_t + G_t$$

This column shows only money gains or losses, all variables are in current money values; the purchasing power of the balance sheet items is not considered since all figures are in current values. (Longer period values are again the mean value for the period.)

Column 6: Real Capital Gains

The consuming unit of Section I is assumed to be influenced only by real variables. The important factor to the consumer is the purchasing power of his resource variables and not the money size of the variables. Therefore, when making an empirical examination of capital gains, real capital gains of the household sector must be derived. These real capital gains, however, cannot be obtained by merely deflating the observations by the proper price index. Capital gains of Column 5 were computed as a residual. Real capital gains should again be computed as a residual; the figure was computed by subtracting real saving for the period from the change in the real value of net worth in the coming period. This procedure not only changes the size of the observation, but can change even the sign of the observation.

$$G_t \text{ real} = \left[\frac{W_t}{P_{t+1}} - \frac{W_{t-1}}{P_t} - \frac{S_t}{P_t}\right]$$

differs from

$$G_t \text{ money} = [W_t - W_{t-1} - S_t]$$

For example, if a person's net worth rose from \$100 to \$110 during the period, and if he had saved \$6 during the period, the money capital gains are \$4. However, if the price level (in this case, the consumer price index) suddenly doubled at the end of the period, then clearly the consumer has suffered a loss. The purchasing power of his net worth in the next period has declined from \$110 to only \$55. His saving for the original period is the same in dollars and is still positive. How can his real net worth decline to only \$55. There have been price movements and the consumer has suffered a capital loss of \$51. (\$55 - \$100 - \$6 = -\$51). While the example is somewhat artificial, the point is clear. It is the stocks that should be deflated according to their purchasing power in the coming period, and real capital gains should be computed from the change in real purchasing power. Real capital gains were computed in this manner and are presented in Column 6. For longer periods, the same procedure was still applied since the actual balance sheets and cumulated saving figures were available. Again the figure for the longer periods is the average value of the period.

Column 7: Liquid Assets

This column is the liquid asset holdings of the household sector at the

beginning of the period. It consists of currency, demand deposits, other deposits (time), short-term government bonds, and government savings bonds. The figures were again obtained from the Goldsmith balance sheets mentioned above. This definition of liquid assets is somewhat arbitrary, but any such definition is arbitrary and will vary with the preferences of the individual. This definition contained the assets which were cash or could easily be turned into cash with little loss. Again the observations for periods longer than a year are the interpolated sums (divided by the number of years) which were derived in the manner outlined above when discussing initial net worth.

Column 8: Age Distribution

This variable is the per cent of population over 65. The figures through 1956 were computed (by dividing the number of persons over 65 by population) from data in Historical Statistics of the United States, Colonial Times to 1957, of the Department of Commerce, Series A 22-23, Table "Estimated Population, by Sex, Color, and Age: 1900 to 1957" (Washington, D.C.: U.S. Government Printing Office, 1960), page 8. The figures for 1957 and 1958 were similarly computed from "Estimates of Population of the United States by Sex, Color, and Age: July 1, 1957-1959," U.S. Government Publications, March, 1960, # 3446, Special Census, Series p-28. Figures for the longer periods are the sum of individual years' observations in the period divided by the number of years. For a discussion of the use of sums, see net worth discussion above, or Part A of this Appendix.

Column 9: Income Distribution

This column is a rough measure of per cent of disposable income received by the upper 5% of the income distribution. The sources are as follows:

Wesley Mitchell, W.I. King, F.R. Macaulay, and O.W. Knauth, Income in the United States, Its Amount and Distribution, 1909-1919 (New York: Harcourt, Brace and Company, National Bureau of Economic Research, Inc., 1921);

Simon Kuznets, Shares of Upper Income Groups in Income and Saving (New York: National Bureau of Economic Research, Inc., 1935), p. 585, observations 1917 through 1948;

Survey of Current Business (April 1959), Table 10, Selma Goldsmith, observation 1957;

Survey of Current Business (April 1960), Table 10, Selma Goldsmith, observation 1958.

Due to the varied sources, a separate series was computed from the source series in an attempt to make the various observations somewhat comparable. It is this series which forms Column 9. Again, the only difference is the period data, each observation of which represents a weighted

(by disposal income) average of the income distribution for the entire period. This figure is again weighted in the final regression as has been discussed above.

Column 10: Consumer Price Index

The figures in column 10 represent the consumer price index with 1958 as the base year. Figures for the longer periods represent the mathematical average of the individual years of the period. These figures do not represent the sum for the period, as this variable is used to deflate the money values of the variables to obtain real values and is not used as an exogenous variable in the regression analysis. The CPI series was obtained from the forthcoming Raymond Goldsmith and Robert Lipsey book cited above, Table 4-1, Part III. Figures there were on a 1929 basis and were shifted to a 1958 basis.

Column 11: Population

This column gives the data on the population of the United States. Figures for the longer periods are the arithmetic average of the figures of the individual years. These data were obtained from the Department of Commerce Publications listed above for Column 8, where population was used to compute the age distribution figure.

C. Regression Results

This part of the Appendix contains Tables 3 through 6 which are the basic regression results—the estimated coefficients and t statistics—in both money and real terms.

TABLE 3

Regression Results: Money Values

$$C_t = B_1 + B_2Y_t + B_3W_{t-1} + B_4G_t + B_5L_t + B_6A_t + B_7D_t$$

18 Observations

Regress Format \ Coef	B_1	B_2	B_3	B_4	B_5	B_6	B_7*
1	12.60 1.34	.873 71.389					
2	18.08 3.23	.605 12.477	.048 5.605				
3	-25.34 - 1.41	.621 14.849	.037 4.336			12.48 2.509	
4	7.40 .82	.891 62.573		- .059 -2.03			
5	19.93 3.01	.575 8.086	.053 4.490	.015 .59			
6	144.31 4.45	.468 10.114	.061 8.738	.033 2.112			-2.89 -3.44
7	-24.02 - 1.20	.612 9.621	.038 3.253	.0045 .199		12.26 2.285	
8	76.92 .85	.843 22.854					-1.436 - .61
9	139.70 3.73	.534 13.488	.051 8.435				-2.931 -3.011
10	89.57 1.07	.856 24.709		- .057 -1.855			-1.994 - .911
11	34.53 2.51	.759 13.335			.099 2.039		
12	28.27 2.21	.783 14.893		- .055 - 2.106	.093 2.11		
13	-27.68 - .31	.749 13.159		- .040 -1.431	.136 2.229		2.036 .772
14	188.90 3.50	.452 8.612	.069 6.571	.039 2.184	- .043 -1.095		-4.298 -2.725
15	176.67 2.24	.462 6.743	.068 5.456	.038 2.096	.046 -1.131	1.439 .207	-4.137 -2.423

* Due to a lack of data, all equations that include this variable are based upon 17 observations.

TABLE 4

Regression Results: Money Values

$$\frac{C_t}{W_{t-1}} = \frac{B_1}{W_{t-1}} + \frac{B_2 Y_t}{W_{t-1}} + \frac{B_3 W_{t-1}}{W_{t-1}} + \frac{B_4 G_t}{W_{t-1}} + \frac{B_5 L_t}{W_{t-1}} + \frac{B_6 A_t}{W_{t-1}} + \frac{B_7 D_t}{W_{t-1}}$$

18 observations

Regress Format \ Coef	B_1	B_2	B_3	B_4	B_5	B_6	B_7*
1	2.59 .34	.886 49.90					
2	6.24 1.32	.650 13.65	.044 5.09				
3	-48.32 - 2.78	.695 17.42	.024 2.68			16.25 3.22	
4	.23 .04	.909 59.17		- .111 -3.42			
5	4.88 .98	.699 9.63	.036 2.94	-.033 -.90			
6	173.16 9.18	.446 18.13	.062 14.13	.043 3.03			-3.53 -7.05
7	-19.18 - 1.11	.647 10.69	.029 2.67	-.006 -.24		14.11 3.88	
8	91.31 .92	.835 18.88					-1.91 - .72
9	171.43 6.97	.521 22.25	.051 15.09				-3.71 -5.72
10	132.54 1.76	.858 25.43		- .118 -3.47			-3.43 -1.69
11	28.39 2.54	.741 13.75			.135 2.79		
12	21.52 2.50	.789 18.53		- .096 -3.56	.110 2.96		
13	43.25 .85	.666 13.92		- .053 -2.07	.185 4.64		.70 .45
14	193.66 6.40	.429 11.32	.070 7.28	.051 2.85	-.032 -.92		-4.26 -4.45
15	157.14 2.83	.478 6.57	.063 4.62	.039 1.54	- .045 -1.07	4.22 .83	-3.84 -3.25

* Due to a lack of data, all equations that include this variable are based upon 17 observations.

TABLE 5

Regression Results: Real Values

$$C_t = B_1 + B_2Y_t + B_3W_{t-1} + B_4G_t + B_5L_t + B_6A_t + B_7D_t$$

18 observations

Regress Format \ Coef	B_1	B_2	B_3	B_4	B_5	B_6	B_7*
1	46.44 1.07	.852 29.50					
2	68.56 2.92	.569 11.64	.048 6.41				
3	91.32 4.28	.467 8.44	.034 4.02			29.91 2.73	
4	15.79 .43	.886 26.16		- .087 -3.07			
5	59.89 2.27	.604 8.96	.044 4.43	-.019 -.75			
6	403.91 2.77	.394 3.74	.051 5.66	.004 .17			-5.37 -2.17
7	85.96 3.48	.489 6.69	.032 3.24	-.010 -.47		29.05 2.54	
8	242.94 .87	.750 5.65					-2.74 - .56
9	389.84 3.29	.408 5.84	.050 8.16				-5.15 -2.48
10	6.52 .03	.877 7.73		- .089 -3.03			.92 .23
11	244.50 3.92	.532 5.83			.185 3.71		
12	174.33 2.56	.636 6.36		- .053 -1.91	.136 2.61		
13	-42.66 - .22	.712 6.38		- .059 -2.23	.147 2.67		4.56 1.26
14	410.96 2.27	.392 3.39	.052 3.77	.005 .18	-.004 -.07		-5.56 -1.51
15	165.97 .90	.446 4.48	.035 2.58	-.008 -.37	- .064 -1.20	38.46 2.32	-1.71 - .49

* Due to a lack of data, all equations that include this variable are based upon 17 observations.

TABLE 6

Regression Results: Real Values

$$\frac{C_t}{W_{t-1}} = \frac{B_1}{W_{t-1}} + \frac{B_2 Y_t}{W_{t-1}} + \frac{B_3 W_{t-1}}{W_{t-1}} + \frac{B_4 G_t}{W_{t-1}} + \frac{B_5 L_t}{W_{t-1}} + \frac{B_6 A_t}{W_{t-1}} + \frac{B_7 D_t}{W_{t-1}}$$

18 observations

Regress Format \ Coef	B_1	B_2	B_3	B_4	B_5	B_6	B_7
1	19.92 .45	.872 19.93					
2	51.61 1.94	.604 10.97	.045 5.53				
3	77.91 3.22	.502 8.50	.026 2.69			33.21 2.73	
4	-15.44 - .42	.913 24.56		- .096 -3.19			
5	33.06 1.12	.664 9.42	.038 3.81	- .036 -1.33			
6	483.80 3.15	.341 2.98	.053 4.14	.006 .22			-6.67 -2.62
7	64.07 2.23	.549 7.04	.023 2.19	-.023 -.93		30.50 2.40	
8	186.59 .68	.783 5.85					-2.38 - .49
9	459.94 4.26	.361 5.30	.052 9.22				-6.29 -3.36
10	-119.80 - .52	.945 8.36		- .108 -3.32			2.63 .66
11	277.06 4.42	.478 5.31			.214 4.63		
12	202.70 2.81	.585 5.64		- .049 -1.77	.167 3.29		
13	36.99 .21	.646 5.44		- .063 -2.32	.168 3.52		3.65 1.24
14	462.22 2.52	.349 2.75	.050 3.29	.003 .11	.015 .26		-6.02 -1.64
15	180.61 1.00	.434 4.04	.031 2.23	-.012 -.53	- .064 -1.16	42.41 2.78	-1.85 - .55

* Due to a lack of data, all equations that include this variable are based upon 17 observations.

Summary: The first part of this study identifies by a priori analysis the determinants of income for college graduates. Methods are developed for measuring these determinants, which are then related to income through a multiple regression equation. In the second part of the study the coefficients of education variables derived from the regression analysis are used to measure rates of return on investment both in graduate education and in improved quality of undergraduate education. Particular attention is directed to changes in computed rates of return caused by explicitly controlling for income determinants which are correlated with education level.

The numerical results are subject to various qualifications, but in general they show that a more careful identification of the income effects which can be attributed to education causes substantial change in computed returns. If the rate of return on more intensive investment in undergraduate education is cut in half, the return on investment in 1 1/2 years of graduate education is raised from minus 100% to about 0%, and the return on 3 1/4 years of graduate education remains unchanged at a low level.

This essay was submitted to Yale University in 1963 in partial fulfillment of the requirements for the degree of Doctor of Philosophy. The author is currently Assistant Professor of Economics at Yale University.

Dissertation Committee: Professors William Fellner (Chairman), John W. Hooper, Charles E. Lindblom, and Raymond P. Powell.

*

I wish to express my appreciation to the members of my dissertation committee, Professors William Fellner and John Hooper, for their encouragement, advice, and guidance. In addition, I have benefited greatly from the comments and ideas of Robin Brooks, James Friedman, Challis Hall, Richard Jolly, Charles Lindblom, and Harold Watts.

The study was carried out while I was in residence at the Brookings Institution and at the Economic Growth Center, Yale University. I am grateful for the financial assistance extended by both these organizations and also by the H. B. Earhart Foundation and the Yale Computer Center.

I also wish to extend thanks for generous assistance received in the data collection tasks required by this study. Among organizations, the Educational Testing Service, Roper Public Opinion Research Center, U. S. Office of Education, National Merit Scholarship Corporation, Center for the Study of Higher Education, and American Alumni Council were particularly helpful. Among individuals, I would like to thank Paul Burnham, John Darley, Frank Fletcher, Richard Levine, H. T. Manuel, and the many college officials who kindly took the time and effort to reply to a questionnaire which I sent them.

S. J. H.

INCOME DETERMINANTS FOR COLLEGE GRADUATES AND THE RETURN TO EDUCATIONAL INVESTMENT

Shane J. Hunt

1. Introduction

In recent years the study of education has acquired renewed popularity, not just because it is a form of expenditure which accounts for some 15% of government outlays,[1] but because it is at least in part a form of investment contributing to economic growth.

This concept of education has led to several studies attempting estimates of the returns from such investment. They have been done partly as guidance to those contemplating further expenditures on their own training, but the more important goal has been to obtain some idea of the efficiency with which society has been allocating its investment funds between additions to the stock of physical capital and to the stock of human skills. The procedure generally followed has been to measure educational rates of return from expected future earnings, as estimated through use of cross section data stratified by age. The rate of return is the discount rate which equates the present values of costs and returns on educational investment. The costs of education consist of the direct costs of operating the schools plus the foregone income of the students who attend them; the return is the difference between the lifetime income flow of the educated individual as it actually is and as it would have been had he not attended school for the period under consideration.

The present study differs from previous work in this field both in the returns measured and in the method of measurement. The first part examines the determinants of income for college graduates, paying particular attention to the influence of education on income. The second part attempts measurement of the rate of return on investment in graduate education and in improved quality of undergraduate education. The method used differs from previous work in two ways. It measures returns which may be attributed to education alone, rather than to a group of correlated factors (e.g., ability, social class) where education is only one element in the group, and it provides confidence intervals for the rate of return estimates derived.

In earlier studies the efficiency of investment allocation was examined by seeing if discounted returns from education exceeded costs when a discount rate equal to the return available from physical investment was used. For example, both Walsh and Friedman and Kuznets used a 4% discount rate without explaining what specific investment alternatives they considered applicable.[2] This 4% rate suggests an alternative available to individual

---

1. Francis M. Bator, The Question of Government Spending (New York: Harpers, 1960), p. 27.

2. J. R. Walsh, "The Capital Concept Applied to Man," Quarterly Journal of Economics, Vol. 49 (February 1935), pp. 255-85. Milton Friedman and Simon

investors, however, rather than the return which society as a whole could obtain through the expansion of business investment. The conclusions of these earlier studies therefore tend to apply more to individual guidance than to social policy. Walsh concluded, for example, that money returns make education for the professional fields of business, law, and engineering profitable to the individual faced with this alternative 4% return, but an M.A., Ph.D., or M.D. are not profitable investments.[1] Friedman and Kuznets used this approach to consider the relative profitability of careers in medicine and dentistry, and found that doctors' incomes exceeded dental incomes by more than the equalizing differential required to offset the longer training period in medicine. They suggested three explanations for this: greater ability requirements in medicine, a shortage of medical training facilities, or restrictive practices which limited the supply of new doctors.[2]

The study of the returns to education moved into the realm of public discussion with the publication of a study by Glick and Miller based on 1950 Census data. Their study reached a widely quoted conclusion that a college education was worth $100,000 to the recipient.[3] This figure represents the increased lifetime flow of income which the authors attributed to college education, but it is not discounted to a present value at the time the costs of college are incurred, and it is not adjusted for mortality. Houthakker reworked the same data, making allowances for discounting, mortality, and federal income taxes, and concluded that with a 3% discount the excess of returns over costs shrank from $100,000 to $25,000. An 8% discount rate reduced it further to $3,300.[4]

The most recent work with this general approach has been done by Gary Becker.[5] Becker's methodology improved on that of his predecessors in three ways. First, he shifted the emphasis of the analysis from personnel guidance

---

Kuznets, Income from Independent Professional Practice (New York: National Bureau of Economic Research, 1946), pp. 147–48.

1. Walsh, op. cit., p. 279.

2. Friedman and Kuznets, op. cit., pp. 118–37; 142–48.

3. Paul C. Glick and Herman P. Miller, "Educational Level and Potential Income," American Sociological Review, Vol. 21 (June 1956), pp. 307–12.

4. H. S. Houthakker, "Education and Income," Review of Economics and Statistics, Vol. 41 (February 1959), pp. 24–28.

5. Gary S. Becker, "Underinvestment in College Education?" American Economic Review, Vol. 50 (May 1960), pp. 345–54. See also the work of T. W. Schultz, "Investment in Human Capital," American Economic Review, Vol. 51 (March 1961), pp. 1–17; and "Education and Economic Growth," in Social Forces Influencing American Education, Nelson B. Henry, editor, The Sixtieth Yearbook of the National Society for the Study of Education, Part II (University of Chicago, 1961), pp. 46–88.

to social policy by comparing the returns from education to the returns obtained by business investment in physical capital. Second, he did not compute the excess of returns over costs at an arbitrarily chosen discount rate but instead computed the discount rate which equated costs and returns. Third, he not only made allowances in his computations for mortality, race, and unemployment, but he also contrived an ingenious although not altogether convincing adjustment which allowed for ability differences between groups of diverse educational level.

There are some rather obvious drawbacks to the foregoing method of estimating the returns to education. Perhaps the most obvious is the implicit assumption, present in all studies except that of Becker, that income differences between, say, college and high school graduates may be entirely explained by the fact that the former went to college. Becker somewhat dampens the severity of the assumption by controlling for race and ability, but obviously there are many more determinants of income than have been mentioned thus far.

The present study explicitly takes account of these various determinants. They are measured simultaneously through the coefficients of a multiple regression equation in which the dependent variable is earned income of male college graduates and the independent variables are those variables which are measurable and which are determinants of income by a priori reasoning. The controlled rate of return on investment in education therefore depends on the coefficients of the age and education variables in this multiple regression. The constraints of data availability restrict the analysis to one particular socio-economic group, college graduates.

Despite a long-standing recognition of the need for such work,[1] there have been few quantitative studies of the determinants of individual income.[2] The literature is even sparser when one turns to the study of income determinants among college graduates, where only two studies can be cited. The first was already mentioned—the work of Friedman and Kuznets; the other, by Havemann and West, is based on the same data which are to be used in the present study.[3] Part of this study will therefore involve reëvaluating the conclusions reached by Havemann and West in the light of different statistical techniques applied to the same data.

---

1. See, for example, George Garvey, "Inequality of Income: Causes and Measurement," Vol. 15, Studies in Income and Wealth, Conference on Research in Income and Wealth, 1952, p. 27.

2. Most such work in recent years has been done at the Survey Research Center. See, for example, their 1960 Survey of Consumer Finances (Survey Research Center, University of Michigan, 1961), pp. 22–23.

3. Ernest Havemann and Patricia Salter West, They Went to College (New York: Harcourt, Brace, 1952).

It should be noted that this proposed improvement to the methodology of computing returns on educational investment by no means eliminates all the difficulties. Before such computed returns can be a reliable instrument of social policy, further research will be necessary to give quantitative estimate to biases arising from several sources. In previous work these biases have been eliminated by making, either implicitly or explicitly, the following assumptions:

1. "Private product, which is the return measured by this approach, is a satisfactory representation of social product, which is the appropriate concern of public policy." Private product is usually defined as the product which accrues to the individual, whereas social product is the aggregate output accruing to all society. Expressed in money terms, the private product of increased education is the increased money income received by those made better educated, and this private return would differ from the increase in net national product if education produces effects reaching beyond those who actually receive the education.

Such effects could work entirely through the economic system, as in the case of widely adopted technical or organizational improvement produced by inventive genius nurtured in the school system, or perhaps as in the case of market imperfections which favor the better educated. Education could also affect social product through its impact on the political and social structure. Social product depends, in the long run, on how well a society is managed, on the wisdom of a host of political decisions ranging from tax cuts to nuclear war. The stock of formal education possessed by a society can certainly affect the wisdom of its collective decisions, although in what way it is not certain. More education may produce greater wisdom, or it may produce political awareness without political wisdom. That is, citizens who were previously apathetic could be transformed by education into partisans of the wrong causes.

This consideration of impact outside the economic sphere points out the inadequacy of setting for education a purely economic goal, such as maximizing net national product. The extent and nature of education helps determine the whole nature of society, and a rational social policy toward education will not attempt to maximize economic return if this goal results in an undesired structure of power, values, or income distribution. For example, an oligarchy may find the returns to increased education to be high in its country and still wish to restrict educational opportunity as a means of retaining power. This contingency demonstrates that a social return to education can be defined only in terms of particular value judgments which define a social welfare function and give weight to the utility levels of the various individuals in society.[1]

1. For further analysis of the social effects of education, see F. W. Garforth, Education and Social Purpose (London: Oldbourne, 1962); A. K. C. Ottaway, Education and Society (London: Routledge and Kegan Paul, 1953).

2. "The rates of return on physical and educational capital chosen for comparison are conceptually similar." From the standpoint of social policy, only Becker's alternative of the return to business investment, the ratio of profits to net worth, is prima facie appropriate, but this comparison has also been challenged. Hansen has noted that this procedure involves comparing an average rate of return on the stock of physical capital to an internal rate of return on college education, and he argues that the former overstates the relative profitability of physical investment by ignoring the time pattern of the return flow from such investment.[1]

3. "The analysis of cross section data provides useful estimates for projecting trends into the future." Much has been written on the dangers of using cross section data for time series projection.[2] In the present instance two factors may be suggested which would tend to bias cross section estimates. First, income differentials at any particular age level are in part determined by the relative supplies of variously educated labor groups. As the per cent of those with a college education increases in each successive cohort group, future income differentials will tend to be smaller than suggested by cross section data. Second, the pattern of technical change in recent decades has tended to shift the demand for labor away from the unskilled and toward the skilled. These two effects work in opposite directions, and the evidence accumulated by Miller suggests that the effects have very nearly offset each other in the case of income differentials between high school and college graduates.[3]

4. "The associated increase in money income is a satisfactory measure of the private returns to education." There are, however, nonmonetary returns, not necessarily positive, which accrue to the educated individual in addition to those described above which could accrue to others in society. These returns include the knowledge and understanding which come from education, the different noneconomic factors surrounding the more educated individual's work, and the enjoyment or misery which the education process itself engenders.

Any evaluation of the net effect of biases introduced by these four assumptions must necessarily be tenuous, even when made in relation to a specific set of value judgments. A set of such judgments is made here, not only because they are subscribed to by the writer but more importantly because

1. W. Lee Hansen, "Rates of Return on Human versus Nonhuman Investment," draft paper, Department of Economics, University of California at Los Angeles, September 1961.

2. See, for example, George Stigler, Trends in Employment in the Service Industries (New York: National Bureau of Economic Research, 1956), pp. 26–46.

3. Herman P. Miller, "Income in Relation to Education," American Economic Review, Vol. 50 (December 1960), pp. 967–69.

they are widely held in the United States and are likely to be accepted by those involved in the process of formulating social policy. These judgments are, first, that power is not too widely distributed in the United States. A greater diffusion of power, while perhaps not needed, is at least not undesirable. Second, the change in values and income distribution which accompanies the expansion of education is not undesirable. Third, knowledge and understanding which produces no economic return is also not undesirable. In addition, the proposition that education does not diminish political wisdom and may increase it is a question of fact rather than values, but it is an article of faith in democracy as well, and so in the policy making process in the United States it is likely to be accepted as true unless proven false.

These value judgments and assumptions all tend to the conclusion that the social return to education is not less than the private return, and is probably greater. In addition, it seems reasonable to conclude, partly from casual observation and partly from the value judgment concerning the value of knowledge, that the total private return exceeds the private money return.[1]

As for the two remaining assumptions, Hansen argues that an internal rate of return gives too low a figure for the returns to education, while Miller's evidence suggests that the use of cross section data does not give substantial bias in either direction. It may be concluded, therefore, that within the framework of these value judgments, the rates of return computed by the method used in this study should represent minimum estimates of the return to education.

In previous studies the additional assumption which virtually eliminates all income determinants except age and education usually introduces an opposite bias. It thus tends to overstate the returns to education, and there results no clear relationship between the computed rate of return and the true social rate of return. The principal advantage of the present method over its predecessors, therefore, is that it attempts to remove ambiguity about the direction of bias. It estimates a lower bound to the social rate of return on investment in education.

2. The Data

The choice of variables to be introduced in the regression analysis depends on the nature of available data as well as on the a priori analysis of income determinants which will be the basis for inferring causation from the regression. Before turning to a description of the variables used, therefore, a brief description of the data is necessary.

For the purposes of this study, a body of data must contain information on a variety of socio-economic variables as well as reasonably precise measures of age and educational level. The only data known to the writer

1. See also T. W. Schultz, "Investment in Human Capital: Reply," American Economic Review, Vol. 51 (December 1961), p. 1037.

which meets these requirements are contained in the Time Magazine College Graduate Survey of 1947.

The sample used in the Time survey was drawn from alumni records of nearly all the four-year colleges and universities in the United States, and thus included college graduates of all ages. Each graduate included in the sample was mailed an extensive questionnaire containing a variety of questions on socio-economic background; slightly over 9,000 replied.[1]

3. The Measurement of Labor Quality

The various determinants of income may be most conveniently separated into three general classes: heterogeneous quality of labor, imperfections in labor markets, and divergence between money and psychic income. Each of these classes will be examined in turn.

Since this study involves only college graduates, who perform little manual labor, variation in labor quality involves variation in mental rather than physical ability. Although mental ability is a multidimensional phenomenon, psychologists have in fact identified a variety of abilities through factor analysis. These abilities are relatively basic and applicable to a wide variety of tasks. They are to a great extent innate, yet capable of being developed by experience. When combined with specific training and occupational experience, these more basic abilities result in a developed capability for occupational performance, but this capability is of economic significance only when combined with personality traits causing the individual to want to use his abilities in an occupational endeavor. These various traits may be collectively termed motivation.[2] Therefore occupational performance requires abilities, as psychologists commonly use the term, motivation, and the highly specific skills which result from formal education and occupational experience.

Mental abilities may be subdivided into cognitive abilities, those involving processes of reasoning, and social abilities, which involve interpersonal

---

1. Further discussion of these data, with particular reference to possible sampling bias, may be found in Shane Hunt, "Income Determinants for College Graduates and the Return to Educational Investment" (unpublished Ph. D. dissertation, Yale University, 1963), Appendix A. See also Havemann and West, op. cit., Appendix, pp. 267–70. A copy of the original questionnaire may be found in Patricia Salter West, "The College Graduate in American Society—A Survey Analysis" (unpublished Ph. D. dissertation, Columbia University, 1951), Appendix B.

2. These traits are referred to as the X-factor by Vernon [Philip Vernon, The Structure of Human Abilities (London: Metheun, 1950)], who describes it as "a somewhat ill-defined factor of industriousness plus interest." (p. 37) Vernon's classification of abilities will be followed here in preference to the Thurstone classification by Primary Mental Abilities solely for the sake of convenience, because Vernon's system emphasizes unity rather than diversity.

relationships. Among cognitive abilities, Vernon identifies two which are of major importance in determining occupational ability in employment which is not of a mechanical or physical nature. One is the so-called "g" factor, an ability which is applicable to and useful for any type of intellectual endeavor. The other, labeled "v:ed," is a somewhat more specific ability applicable to verbal, numerical, and educational pursuits.[1]

The distinction between cognitive and social abilities is by no means clear-cut, since many abilities commonly described as social involve substantial elements of cognition. Social abilities involve perception and communication, that is, the ability to recognize and predict the psychological states of others and the ability to communicate one's acts or feelings in accordance with the expectations of others.[2] Both of these abilities involve cognition. The ability to recognize and predict the psychological states of others, sometimes called empathic ability, depends partly on understanding produced by knowledge and experience. Similarly communication may or may not be cognitively controlled; if communication is a cognitive process it is dramatic ability. Social abilities become determinants of income in any occupation which involves either making decisions about the attitudes of other people or persuading other people.

How may these components of labor quality be measured? Turning first to cognitive abilities, a useful, and measurable, proxy variable is available in the form of the individual's record of academic achievement, i.e., his grade average. Both educational ability and job ability consist largely of the factors "g" and "v:ed," as modified by motivation, and thus "the structure of job abilities appears to parallel closely that of educational abilities."[3] This makes academic grade average a better measure of realized job ability than an IQ score would be, since intelligence tests involve the "g" factor alone, and contain no elements of "v:ed" or motivation.[4]

To be more explicit, the proxy measure of cognitive abilities which will be used in this analysis is the overall grade point average achieved by the

---

1. Ibid., p. 23.

2. Urie Bronfenbrenner, et al., "The Measurement of Skill in Social Perception," in Talent and Society, David McClelland, editor (Princeton, N. J.: Van Nostrand, 1958), p. 97.

3. Vernon, op. cit., pp. 126–27.

4. Ibid., p. 64. For other discussion of the close connection between academic achievement and occupational achievement in later life, see Douglas Bray, Issues in the Study of Talent (New York: Kings Crown Press, 1954), pp. 38, 45; and Fred Strodtbeck, "Family Interaction, Values, and Achievement," in Talent and Society, p. 243. McClelland points out, however, that the relationship is by no means perfect. See David McClelland, "Issues in the Identification of Talent," in Talent and Society, p. 3.

individual during his four undergraduate years in college. Before these averages can be used in a regression equation, however, adjustments must be applied to account for the fact that grading standards are by no means uniform among the various colleges and major fields represented in the Time data.

Adjusting Grade Averages among Major Fields

These adjustments will be accomplished through tests of scholastic ability given to students as they either leave high school or enter college. There are many such tests in use; all measure the same types of ability, but all are scored differently. Fortunately we are not confined to one test in this series, since Darley has prepared conversion tables by means of which several such tests are reduced to a common scale.[1]

We may first consider the problem of adjusting for variation in grading standards among different major fields within a single given college. There is a set of regression equations:

$$T_i = a_i + bG_i + u_i, \qquad (1)$$

where T_i = test score, and G_i = grade average in the i^{th} major field.[2]

This equation incorporates two assumptions which require verification: a linear relation between test scores and grades, and a coefficient of grades which is the same in all major fields (i.e., the grade coefficient, b, has no subscript). Since no test score information is contained in the Time data,

1. John Darley, et al., unpublished manuscript, Center for the Study of Higher Education, Berkeley, California, Appendix C. The best-known tests in this group are the American Council on Education Psychological Examination and the Scholastic Aptitude Test of the College Entrance Examination Board.

2. At first glance this may seem a perverse formulation, since causal and temporal sequences are reversed. It says that test score, acquired at the beginning of college, is a function of college grade average, determined four years later. It should be remembered, however, that college grades are a proxy for an ability mix possessed by the individual long before graduation from college. The objection might also be raised that this formulation would result in severely distorted correlation and regression estimates, because the dependent variable is a criterion for admission to college, and thus its lower tail would be truncated. [See Harold Gulliksen, The Theory of Mental Tests (New York: John Wiley, 1950), pp. 128–32]. However, it should be remembered that test scores are only one criterion for admission to college, and so the use of these scores in refusing applicants is a weeding process, not a truncation. Further, there is no assurance that the distribution of applicants' test scores is exactly normal. If it is positively skewed, weeding by means of test scores would push the test score distribution of admitted students toward normality.

an examination of these assumptions required collection of another body of data, consisting of paired observations of grade averages and test scores. Each observation pertains to an individual, and the data include all members of a recent graduating class in each of ten different colleges and universities.[1] Regressions computed for 3 of these 10 colleges showed that the linearity assumption could not be rejected at a 5% significance level in 18 of the 21 major fields examined. In addition, the hypothesis of similar slopes among major fields within a given college was tested for all 10 colleges and was rejected at the 5% level in only one of the ten. It was therefore concluded that the assumptions implicit in equation (1) are not unreasonable for what will admittedly be a rough adjustment process, even when the adjustments are applied not just to these 10 colleges but to American colleges in general.

The procedure which makes use of equation (1) consists of deriving a series of adjustments which will be added to each individual's grade average to derive an estimate of the average he would have received if he had majored in humanities or social sciences. This procedure may be visualized as a process in which observations are shifted horizontally, so that all major field regression lines within a given college become coincident. One major field, in this case humanities and social sciences combined, provides the standard to which others are adjusted. The regression line of the standard field is labeled A in Figure 1, while B is the regression line for another field. To make these lines coincident, $(G_A - G_B)$ must be added to every observation in field B. This is the same as $(T_B - T_A)/b$, i.e., the difference in intercepts divided by the common slope.

The information was not available to make these adjustments separately for every college represented in the Time data. Although the adjustment required for any particular field varies among colleges, nevertheless there was sufficient similarity in the pattern of adjustment factors in the data from the 10 colleges to permit the derivation of a general pattern of adjustments, applicable to any of the 10 colleges, and by extrapolation, to colleges in general. Applying this general pattern, which is shown in Table 1, to any particular college will not eliminate the existence of divergent grading standards, but it should reduce them. These adjustments show, for example, that in general a graduate in the field of education would have received a grade average 0.6 points lower (where A = 3.0, B = 2.0, etc.) if he had instead majored in humanities or social science.

---

1. A more extensive description of this data and the various tests applied to it may be found in Hunt, op. cit., Appendix B.

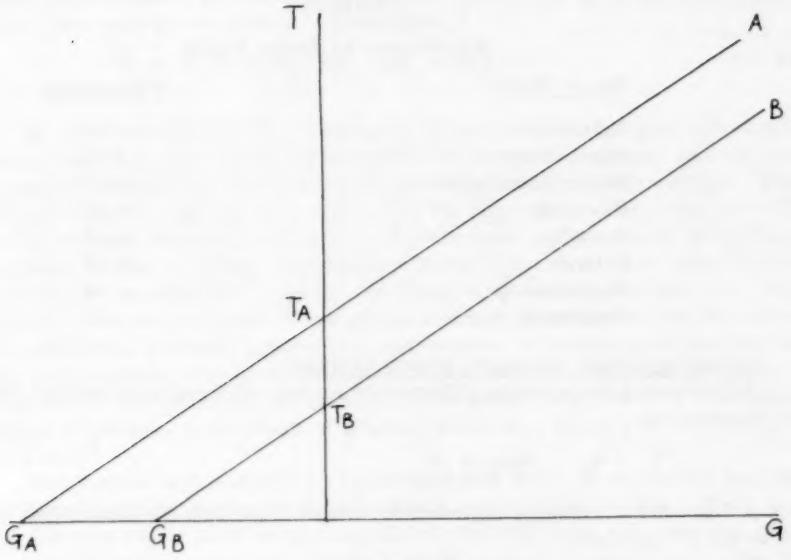


Figure 1

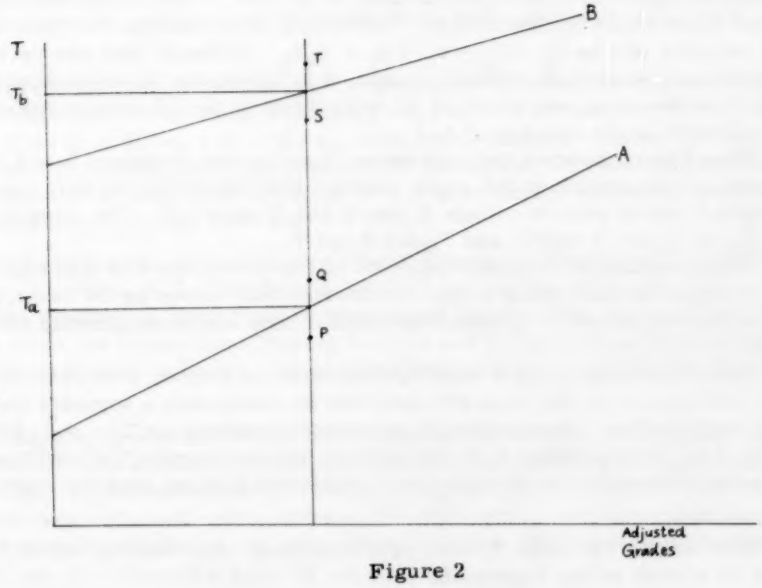


Figure 2

TABLE 1

Adjustment by Major Field

Major Field	Adjustment
Education	-.6
Agriculture	-.6
Home Economics	-.4
Business	-.2
Nursing	-.4
Science	+.2
Engineering	+.4
Pharmacy	-.6

Adjusting Grade Averages among Colleges

Adjustments for grading differences among colleges are derived through a set of equations:

$$T_j = a_j + b_j A_j + U_j \qquad (2)$$

where A = grades previously adjusted for major field and the subscripts now refer to the j^{th} college.

Since the scale by which adjusted grades are measured is an arbitrary one, this variable may be rescaled through any linear transformation without affecting its significance as a determinant of income. A set of transformations based on equation (2) is chosen here. Within any given college the transformed ability variable will be $\tilde{T}_j = T_j - u_j = a_j + b_j A_j$. Although this new variable is therefore a measure of adjusted grades, it is expressed in terms of test scores; it is the mean test score of all individuals in the j^{th} college possessing an adjusted grade average of A_j.[1]

Thus Figure 2 shows the regression lines for two colleges, A and B, and points corresponding to the grade average and test score for four individuals, where P and Q went to college A, and S and T went to B. The computed value, $\tilde{T}_j$, is T_a for P and Q, and T_b for S and T.

The advantage of this particular set of transformations is that adjusted grade averages for all colleges are transformed into values on the same test score scale, and therefore grades from all colleges are made directly comparable.

This adjustment method must operate with the further constraint that neither sufficient time nor data are available for computing a separate regression for each college. In equation (2) we therefore substitute $\bar{T}_j - b_j \bar{A}_j$ for a_j, and $r_j s_{T_j} / s_{A_j}$ for b_j, where r_j is the product moment correlation coefficient. These substitutions derive directly from the nature of least squares regression

1. In other words, the mean Y value of a conditional distribution, where X is fixed, is a point on the regression equation $Y = f(X) + u$.

estimates and require no assumptions about the normality of the distribution of either test scores or grades.[1] Therefore:[2]

$$\tilde{T}_j = \bar{T}_j + (r_j S_{T_j} / s_{A_j})(A_j - \bar{A}_j). \qquad (3)$$

By this formulation $\tilde{T}_j$ is estimated from the adjusted grade average for each individual, the correlation coefficient for each college, and the means and standard deviations of the marginal distributions for each college. The individual's grade average is available in the Time data. This data also identifies the college which he attended, and the means and standard deviations of test scores for each college represented in the Time data have been obtained from a variety of sources.[3] As for the remaining three variables, i.e., the mean and standard deviation of the grade average distribution and the correlation coefficient between grades and test scores, it is assumed that they are reasonably constant among American colleges, and the "ten college" data is used to derive the following estimates: correlation coefficient, 0.5; standard deviation of grades, 0.45; mean of grades, between 1.6 and 1.9, depending on major field.[4]

One further adjustment is required of this data. It should be remembered that the grade averages of individuals in the Time survey reflect academic careers undertaken at various times over the past half century. The mean test scores all pertain to a single point in time, however, viz., the class which entered these various colleges in 1952 and graduated in 1956. As the

---

1. J. Johnston, Econometric Methods (New York: McGraw-Hill, 1963), pp. 11, 30.

2. This is similar to the common scaling procedure which assumes equality of standard deviates. In an equation $y = a + bx$, $r = 1$, then $y = \bar{y} + (s_y / s_x)(x - \bar{x})$ or $(y - \bar{y}) / s_y = (x - \bar{x}) / s_x$. See John Flanagan, "Units, Scores, and Norms," in Educational Measurement, E. Lindquist, ed. (Washington: American Council on Education, 1951), p. 727. For a similar problem, see also Gulliksen, op. cit., pp. 296–304.

3. College Entrance Examination Board, Manual of Freshman Class Profiles, 1961 (Princeton, N.J.: The Board, 1961); John Darley, et al., op. cit.; Dael Wolfle, America's Resources of Specialized Talent (New York: Harpers, 1954). Also from the Educational Testing Service and from mail questionnaires sent to individual colleges. For further information see Hunt, op. cit., Appendix C.

4. These variables did in fact show a significant degree of variation among the 10 colleges studied, and the assumption of consistency therefore quite evidently introduces error into the resulting measure of ability. However, the error does not appear to be systematic. The ability estimates thus derived are therefore imperfect, but not strongly biased. For further discussion see Hunt, op. cit., Appendix B.

adjustment procedure now stands, there exists an implicit assumption that the quality of education has remained constant over time. Such is not the case, however; educational quality has probably improved over time, simply through the expansion of knowledge if for no other reason. This trend should be reflected in a secular change in grading standards, and is represented in Figure 3, which depicts the rising trend in mean ability levels, as measured by test scores, at various colleges. Each curve in the upper half of the diagram relates mean test scores for classes graduated in an earlier period to mean scores for the class of 1956. The oldest period is represented by curve A, and as time passes the curve rises, until it eventually merges with the 45 degree line. The lower half shows how the relation between present (i.e., class of 1956) test score and income also shifts over time. Points A and B represent the same real ability level, achieved at different times in the past. Although they are reflected in different present test score levels F and G, they still have the same influence on income, so the curves relating present test score and income must be spaced so that points C and D are on the same income level.

The family of curves in the upper half of Figure 3 is given by:

$$\bar{T}_z = c_z + \bar{T}_p + u_z,$$

where $\bar{T}_z$ = mean test score for a college in the year z, and $\bar{T}_p$ = mean test score in 1956. For simplicity we may postulate an even rate of change over time in the position of the curve, such that $c_z = -e(1956 - z)$. Therefore:

$$\bar{T}_z = -1956e + e_z + \bar{T}_p + u_z \qquad (4)$$

$$= -9e - eY + \bar{T}_p + u_z,$$

where $Y = 1947 - z$, the years elapsed since graduation.

Since the $\bar{T}$ referred to in equation (3) is actually $\bar{T}_z$, that is, mean test score of the college not in 1956 but when the individual actually graduated, we substitute equation (4) into (3) and get as a measure of cognitive ability:

$$\tilde{T}_j = (\bar{T}_{pj} + (rs_{T_{pj}} / s_{A_j})(A_j - \bar{A}_j)) - 9e - eY + u. \qquad (5)$$

Before leaving the problem of measuring cognitive ability, some implications of this measurement procedure should be noted. First, the test scores which have been used for adjusting grade averages between both major fields and colleges do not measure the same complex of abilities that grade averages do. They measure both "g" and "v:ed," but probably do not cover the personality factors of motivation. Therefore if one college or major field contained students particularly motivated for academic—and occupational—achievement, their ability measures would be biased by this adjustment method. Second,

there is further inaccuracy which arises from using a single variable to measure cognitive abilities, since, although nearly all occupations and collegiate courses of study require the same cognitive abilities, they require them in different proportions. This is particularly evident in the case of verbal and quantitative components of "v:ed"; scientific and engineering fields obviously require greater relative weighting of quantitative ability.

These difficulties are partly avoided by eliminating those occupations requiring particularly unusual combinations of abilities, i.e., music and art, but the remaining variety in required ability mixes cannot be considered with available data, and must be disregarded. The test score used is the sum of the verbal and quantitative components of the tests. It should be noted, however, that scores on these component parts of the various tests are positively correlated, with an r of .6 to .7, and this reduces the error introduced by nonoptimal choices of weights.[1]

Finally, it should be noted that the curve relating mean test scores in 1956 and in earlier years, depicted in the upper half of Figure 3, is assumed to have maintained a constant slope over time. This is the same thing as saying that ability differences between colleges have remained constant. Such an assumption is probably not valid for the past decade, when ability levels in the high-quality colleges have risen remarkably. The assumption may be quite valid for the half century prior to 1947, the year of the Time survey, however. If the slope of the curve has changed, another variable, a cross product between years since graduation and ability, would have to be added to equation (5).

Other Aspects of Labor Quality

Measurement problems are even more difficult when one turns from cognitive to social abilities. There does exist in the Time data, however, information which can be used to derive a proxy measure for social abilities. This is the extent of the individual's participation in collegiate extracurricular activities. It must be noted, however, that the attributes required for successful participation in extracurricular activities have never been identified with any precision. It seems reasonable to expect this measure to reflect a composite of social and cognitive abilities, plus motivation. As such it provides a useful complement to grades, since it reflects attributes applicable to occupational performance which, due to the particular values of the individuals involved, may not have been applied to scholastic achievement.

How is the variable to be quantified? Several attempts have been made to measure such participation by setting up more or less arbitrary numerical

1. Educational Testing Service, A Summary of Statistics on Selective Service College Qualification Test of December 13, 1951, April 24, 1952, May 22, 1952 (Princeton, N.J.: ETS, 1953), Appendix 2. See also John T. Cowles and Marjorie A. Olson, "The Verbal-Quantitative Balance of the 1951–54 Selective Service College Qualification Test," School and Society, Vol. 80 (November 27, 1954), pp. 169–73.

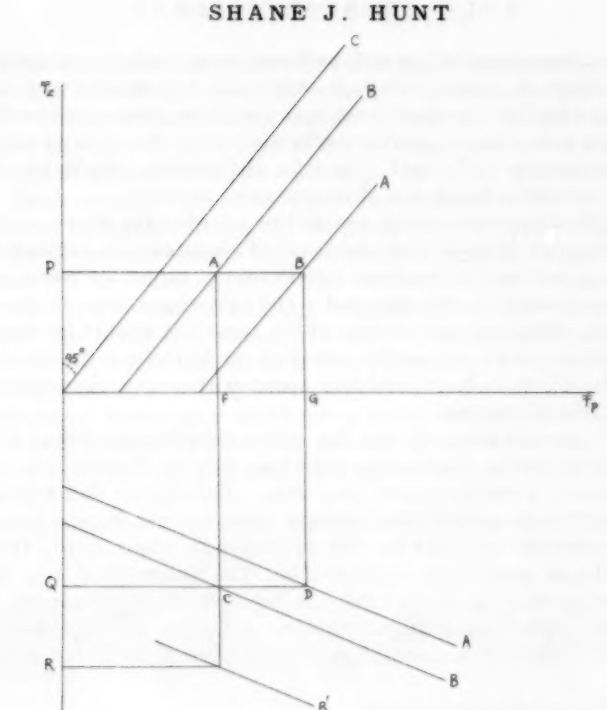


Figure 3

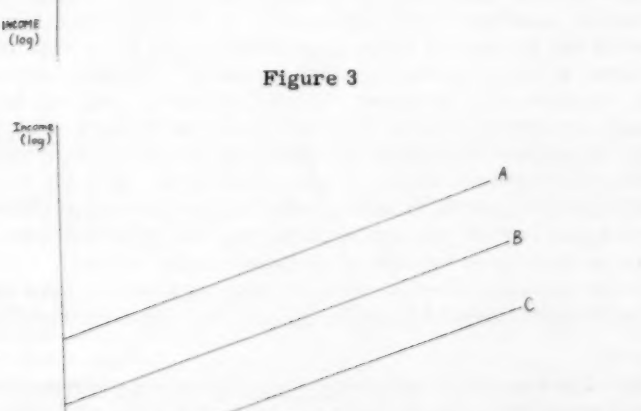


Figure 4

scales.[1] Although these scales all seem to assign similar weights to the same activities, the actual scale construction is usually described in such vague terms that it was impossible to relate scale values to the coded data in the Time survey. An exception to this tendency toward vagueness is the study by Roskens, which presents a numerical value, as decided on by a panel, for 50 different activities at the University of Iowa. From these figures values were derived for membership and officeholding in the activity groups coded in the Time data.[2]

This procedure involves a great deal of extrapolation, since values obtained at a single university are applied to all colleges in the country. This implies, first, that there is constancy among colleges in the rankings of various activities. This is not an unreasonable assumption; similarity in the structure of extracurricular activities at most colleges is an easily recognized phenomenon. Second, it implies that the mean level of the ability mix measured by these activities is proportional to the extent of such activities at any particular college. Thus this measure is analogous to a grade average unadjusted for differences in major fields or colleges. Unlike the case of grade averages, there are no standardized scores available for adjustment, and so this unstandardized measure must be used.

Further difficulty arises from possible secular change in the nature and extent of extracurricular activities, a phenomenon which casts doubt on direct comparisons of scores assigned individuals of different age. To deal with this problem, we postulate that institutionalized extracurricular activities have become a larger part of student life during the past half century, with the result that a given level of participation has become indicative of progressively lower levels of the ability mix which this measure represents. This hypothesis is depicted in Figure 4, where the regression line declines over time from A to C. This family of curves is represented by the equation:

---

1. Bessie Lee Gambrill, College Achievement and Vocational Efficiency (New York: Teachers College, Columbia University, 1922); D. S. Bridgman, "Success in College and Business," Personnel Journal, Vol. 9 (June 1930), pp. 1–19; L. F. Carter and M. Nixon, "An Investigation of the Relationship between Four Criteria of Leadership Ability for Three Different Tasks," Journal of Psychology, Vol. 27 (January 1949), pp. 245–61; Victor Jepsen, "College Activities and Vocational Success," Occupations, Vol. 29 (February 1951), pp. 345–47; Joseph Fink, "Collecting and Quantifying Social Participation Information," Personnel and Guidance Journal, Vol. 36 (February 1958), pp. 417–19; Ronald Roskens, "The Relationship between Leadership Participation in College and after College" (unpublished Ph.D. dissertation, Department of Education, State University of Iowa, 1958).

2. A more complete description of this extracurricular activity index may be found in Hunt, op. cit., Appendix C.

$$\log I_z = d_z + fX + v_z$$

where I = earned income and X = index of extracurricular activities. Assuming an even rate of change over time, $d_z = e' + e''(Y)$, so that as before, the shift over time influences the constant term and the years since graduation variable in the multiple regression. If the slope as well as the intercept of the curves shown in Figure 4 has shifted over time, a cross product of years since graduation and extracurricular activities should also be added.

Education improves the quality of human labor, and is transmitted to the individual not only through formal schooling, but also in his family upbringing and subsequent work experience. The individual's pre-college training, both in home and in school, should be reflected in his academic and extracurricular achievements in college as these achievements are measured above. The mean quality of education offered by his college is not so reflected, however, since the test scores used for adjusting grade averages were taken at the beginning of college. If, for example, we compare freshman classes at colleges of significantly different quality, we may expect that, even if these groups showed identical mean ability levels as freshmen, superior college quality would produce superior mean ability levels four years later as well as higher incomes in later life. Such ability differences have not been accounted for by the measure of adjusted grades explained above. In a study of investment in education we are not directly concerned with college quality, however, but rather with the more basic inputs which constitute the physical resources devoted to the education process. The money value of these resources, expenditure per pupil, is introduced as another determinant of labor quality. The data for this measure[1] relate to a single academic year, 1953–54. To account for secular change in expenditure per pupil, an adjustment must be made similar to that applied to mean test scores and depicted in Figure 3.[2] This means that expenditure per pupil will be represented in the final regression equation by contributions to the years since graduation variable, the constant term, and the error term as well as by a separate variable of its own. In addition, there will be a cross product of years since graduation and expenditure per pupil if the slope of the curves in the lower half of Figure 3 changes over time. Also, it should be recognized that educational quality is not a linear function of expenditure per pupil because many colleges are too small to spread their overhead costs effectively,

1. The derivation of this data is explained in Hunt, op. cit., Appendix C.

2. The axes measuring present and past mean test scores now measure present and past levels of expenditure per pupil, both expressed in 1953–54 prices. Also, there now exists the possibility that technical change will cause a shift over time in the relation between income and expenditure per pupil. Thus expenditure level P which results in an income Q in period A may result in higher income R in later period B.

so a size of college variable, measured by student enrollment, must also be introduced. And finally, the relationship between expenditure and quality may be expected to be quite different for Catholic colleges, where the salaries of teaching members of religious orders are very low compared to those received by their secular counterparts in other colleges. Therefore a dummy variable, coded 1 for Catholic schools and 0 for other schools, is also introduced.

Among college graduates there exists educational variation in the extent of their graduate training as well as the quality of their undergraduate colleges. The extent of graduate training is measured in the traditional manner, by the number of years of such education, and therefore the measure, as derived from the Time data, suffers from two shortcomings. First, no attempt is made to account for quality differences in graduate education. Second, the Time data record only graduate and professional degrees, so graduate work not culminating in a degree passes unnoticed. This measure, like others described above, therefore contains imperfections, but is still quite meaningful.

Experience gained from work is the main reason that income is an increasing function of age in the early stages of the life cycle. The amount of knowledge acquired through work experience depends more directly on the number of years the individual has worked in positions closely related to his present occupation, and it seems reasonable to suggest that the number of years elapsed since graduation, a figure available in the Time data, is at least as good a measure as age and may well be better. This is the variable which may be influenced by secular change in test scores, expenditure levels, and extracurricular activities.

An individual's work experience which is pertinent to his present employment may be less than indicated by the number of years since graduation if at some time he has made drastic changes in his occupation. In 1947 the most important such change had been made by those who were veterans of World War II. Not only did they lose years of potential work experience; many had not achieved complete readjustment to the civilian world by 1947. For this reason an additional variable is introduced, a dummy variable coded 1 for veterans and 0 for nonveterans.

To summarize, the following variables will be considered as determinants of income, each measuring an aspect of labor quality.[1]

1) $\bar{T}_p + (rs_{T_p} / s_A)(A - \bar{A})$ — referred to subsequently simply as "ability."—(Q).

2) Extracurricular activity indix—(X)

3) Expenditure per pupil—(E)

4) Enrollment—(N)

1. For an account of the method by which all variables were quantified, see Hunt, op. cit., Appendix C.

5) Catholic college dummy variable— (C)

6) Years of graduate education —(J)

7) Years elapsed since graduation —(Y)

8) World War II veteran dummy variable— (V)

4. The Measurement of Other Income Determinants

Market Imperfections

Imperfections in labor markets may be caused either by monopoly elements or by imperfect knowledge. Among phenomena which can be classed as monopoly elements, there is little evidence to suggest that market control exercised by buyers of labor services (e.g., monopsony) is an important determinant of income for college graduates. Market control exerted by sellers of labor cannot be similarly dismissed, however, since in several occupations the practitioners control entry by setting standards of admission.[1]

Monopsony and restrictionism are controls which may be applied uniformly to all offerers of labor services. Controls applied on a discriminatory basis, however, are probably of greater importance for college graduates. Advantages in the job market may result from the possession of personal qualities which are valued by the employer even though they contribute nothing to occupational performance. Such personal qualities are largely associated with social class, race and religion being here considered aspects of social class. Advantage in the job market also results from the first of two aspects of imperfect knowledge which will be considered here, i.e., ignorance of the full range of opportunities in the labor market.[2] Because of such ignorance, access to job opportunity becomes a determinant of income, the advantage lying with those job seekers who happen to be known by the employer, as a result of either special influence exerted through family or friends, or merely of luck.

Second, there is employer ignorance in evaluating the qualifications of job applicants, i.e., in predicting future labor productivity. The employer must hazard predictions from the applicant's past record in school or work,

1. The medical profession in particular has been accused of restricting supply to improve medical incomes as well as the quality of medical practice. See Friedman and Kuznets, op. cit., pp. 12, 135–37; Seymour Harris, The Market for College Graduates (Cambridge: Harvard University Press, 1949), pp. 134–43.

2. For a discussion of costs and returns involved in expanding knowledge of job opportunities, see George Stigler, "The Economics of Information," Journal of Political Economy, Vol. 69 (June 1961), pp. 213–25; and "Information in the Labor Market," Journal of Political Economy, Supplement, Vol. 70 (October 1962), pp. 94–105.

but this record contains only proxy measures which are, for an individual, rather poor predictors of labor productivity. These proxy measures are evaluated according to the occupational performance of those already employed who possess the same attributes, and so the job applicant finds himself judged not by the labor productivity which he possesses, but by the average productivity possessed by groups with which he is associated through these proxy measures. A college graduate, therefore, may have great advantages over nongraduates not merely because he is abler, but because he is identified as a part of an abler group. The same is true of the applicant who may be identified with a relatively high social class or a particularly prestigious college. This system of selection is an efficient one, provided the measurements used are derived from the most homogeneous groups with which the applicant is associated. A preference for using measures from relatively heterogeneous groups is both inefficient and discriminatory.[1]

Thus discrimination, either as an intended policy or as a consequence of imperfect knowledge, influences income levels; and social class, the principal criterion of discrimination, becomes a determinant of income. There are proxy measures for social class in the Time data, but they are rather tenuous. The principal social class variable used by Havemann and West was the percent of college expenses earned by the individual while he was in college.[2] The Time data also offer an alternative variable which tells whether or not the individual's parents attended college, and as a final social class variable, the individual's race is also recorded.[3]

Divergence between Money and Psychic Income

Psychic income is by definition the object of maximizing behavior. Any nonmonetary determinant of psychic income which is not present in equal magnitude in all occupations would tend to give rise to compensating variation in earned money income and would therefore become a determinant of earned money income. The variables discussed thus far are determinants of wage rates rather than psychic income, however. Wage rates are of course a principal determinant of psychic income, but they are several links removed. Wage rates and leisure preferences combine to determine earned money income; earned money income must be deflated by a geographical price index

1. For instance, a given college academic record defines a relatively homogeneous group and should be used as a predictor of future productivity in preference to the more heterogeneous groups defined by race or religion.

2. Havemann and West, op. cit., Chapter 14. Also Patricia Salter West, "Social Mobility among College Graduates," in Class, Status, and Power, R. Bendix and S. Lipset, editors (Glencoe, Ill.: Free Press, 1953), pp. 465–80.

3. Religion was included in the original Time data, but this variable was not included on the card decks which were that part of the Time data obtained for use in this study.

to equal real earned income; finally various other effects must be added to real earned income to arrive at psychic income. Among these other effects, four types may be identified:

1) Goods and services for which some must pay, but which are furnished without money payment to other individuals. Perquisites, fringe benefits, and production for home consumption fall within this category.

2) Money costs of earning income. This includes both costs of commuting and of maintaining a consumption level required by one's occupation if this exceeds preferred consumption.

3) Other sources of utility not associated with the work environment. These are of infinite variety, but for simplicity we may note geographical variation in the physical environment, involving climate, scenery, etc., and variation in the human environment, involving access to cultural activities and the nature of social patterns.

4) Other sources of utility associated with the work environment. These sources show occupational rather than geographical variation, and among them we may mention degree of boredom, status, physical discomfort, personal satisfaction, and risk.

The Time data give no help in moving from wage rates to real earned income, because they contain no information on the number of hours worked per week, and, as far as could be ascertained, there is no information anywhere on price level differences between regions or between cities of different size.[1] The picture is brighter with regard to the factors other than real earned income which are determinants of psychic income. None of the available variables measure one effect in isolation; however they all involve some combination of effects. City size may be expected to be a determinant if it is correlated with price levels or the money costs of earning income (especially commuting costs).

Geographic region could be anticipated as a determinant of income mainly through lags in the adjustment of the labor force to changing geographic patterns of labor demand. In additon, however, regions may have different price levels, and different psychic income from the physical environment, particularly climate.

The principal variable for measuring nonmoney income associated with the work environment is of course the individual's occupation, but like the preceding variables occupation can reflect other factors as well, viz. restricted entry and quasi-rent caused by unforeseen shifts in supply and demand. The following principle is proposed for occupational stratification: each occupational group should be reasonably homogeneous with regard to these three causes of differentials and should be chosen so that a customary

1. The Bureau of Labor Statistics, U.S. Department of Labor, publishes cost of living comparisons for only a few of the largest cities, as in their Family Budget of City Worker, October 1950, Bulletin 1021 (Washington: U.S. Government Printing Office, 1951).

progression of jobs throughout life does not involve a switch from one group to another. For example, if the normal path of promotion leads salesmen to become executives, then salesmen and executives should be in the same occupational group, even if they do receive different amounts of nonmoney income. This suggested principle argues against an over-fine classification of occupations. It is advanced as a means of getting the most out of an occupational variable without resort to an over-fine classification which makes occupation collinear with ability or age.[1]

In the present analysis five occupations will be considered. Medicine and law are treated separately because of the possible presence of restrictionism and the special status accorded members of the professions. In addition, medicine may deserve special treatment both because of its humanitarian purpose and its heavy work load. Government and education are accorded separate treatment because there may be greater security of employment or greater sense of social purpose. The fifth occupation is all business. Although a division between technical and administrative tasks would have been preferred, provided the path of promotion does not lead from one to the other, this was not available in the Time data, and so business remains undifferentiated.

One final variable to be considered distinguishes between the salaried and the self-employed. Earned income differences between these groups should be attributable to risk premiums, but it should be remembered that in part they may represent a return to capital which is poorly differentiated from labor earnings.

To summarize again, the following variables will be considered as determinants of income in addition to those listed at the end of the preceding section:

1) College self-support—(S_p)

2) Parents' college attendance dummy variables—omitted category: neither parent.

   a. Father only— (F)
   b. Mother only — (M)
   c. Both parents — (P)

3) Race dummy variable— (R)

4) City size

   a. Dummy variable coded 1 for cities under 1,000,000, otherwise 0 — (0)

1. For example, the study by Thorndike and Hagan used a very detailed classification of occupations and concluded that, within each occupation, there was little or no relation between ability and income. Robert Thorndike and Elizabeth Hagan, Ten Thousand Careers (New York: John Wiley, 1959).

b. Population of cities under 1,000,000 — (Z)

5) Occupation dummy variables

a. Business — (B)
b. Medicine — (M_d)
c. Law — (L)
d. Government — (H)
e. Education — (U)

6) Region dummy variables — omitted category: North

a. West — (W)
b. South — (S)

7) Self-employed dummy variable — (K)

5. Regression Computations

The possible determinants of earned income available in the Time data have thus been identified a priori. There is no a priori reasoning however, which can suggest the functional form by which these variables should be related to income in a regression equation, and so a pilot sample was drawn from the Time data for this purpose. The pilot sample consisted of every fifth observation, 661 in all, and was used to determine which variables to include in the final regression equations as well as the form in which they are to be included.

The Pilot Study

The pilot study followed the following procedure. First, a function relating income to years since graduation was fitted by using a series of linear segments to approximate the nonlinear relationship. Next, the variables enumerated above were added to the regression equation. If their regression coefficients differed from zero at the 5% level of significance, using a two-tailed test, the variables were retained in the equation; if not, they were discarded. Then after going through the full list of variables in this manner, those which had been discarded were added back one at a time to make sure they were properly discarded. Alternative formulations of the ability variable were also introduced to test the effectiveness of the particular variable described above.

The pilot study then turned to the cross products, which were added either singly or in small groups and were retained in the regression if significant. In some cases the addition of a cross product caused a previously significant simple variable (i.e., a variable which does not involve a cross product) to become insignificant. The question then arose, which formulation should be chosen, the significant simple variable without cross product, or the significant cross product without the simple variable? The criterion

used for an answer was to choose that formulation which had the higher R^2.

After the cross products chosen for examination were evaluated in this manner, one final review was undertaken, in which all the rejected simple variables were added one at a time to the equation. When they were all found to have coefficients which were insignificantly different from zero, the appropriate regression form and the appropriate variables were considered to be found and the pilot study was ended.

The piecewise linear function relating income to years since graduation was of the following form:

$$\log I = a_0 + a_1 Y + a_2 D_1(Y-6) + a_3 D_2(Y-16) + a_4 D_3(Y-26) + a_5 D_4(Y-36) + u$$

where earned income, I, is expressed in logarithmic form to reduce skewness in its distribution. This is done to better meet the requirements of regression analysis that the error term be both normally distributed and of the same variance in all possible conditional distributions.[1] Y = years since graduation and the D's are a series of dummy variables coded 1 if the adjacent expression in parenthesis [(Y−6), (Y−16), etc.] is positive, 0 if negative. The coefficient a_1 gives the slope of the function on the first linear segment, between Y values of 0 and 6; subsequent coefficients a_2, a_3, a_4, and a_5 show the change in slope occurring at the bending points in the function where Y equals 6, 16, 26, and 36 respectively.

When only simple variables were considered in the regression, the variables found to be significant at the 5% level were years since graduation (Y), the variables which make this function bend at 26 and 36 years, self-employment (K), ability (Q), and the dummy variables for medicine, government, and education. This is shown as regression 1 in Table 2. The t statistic is the computed value of the regression coefficient divided by the standard error of the coefficient.

The alternative formulations of the ability variable were introduced one at a time into regressions containing all the variables of regression 1 except medicine. The results are shown in Table 2 under the title Ability Regressions. Even unadjusted grades (1), with a t statistic of 3.18, is seen to be a significant determinant of income, but adjusted grades (2) are more significant, and the formulations which introduce college mean test scores (3, 4, 5) are better still, using R^2 values as criteria for evaluation. Among the three formulations involving $\bar{T}$, there is no difference among them by the R^2 criterion, and (5) is chosen because (3) and (4) really represent the same thing split randomly into two components. Or to put the advantage of

1. Johnston, op. cit., pp. 9, 20. This particular transformation is chosen because income tends to approximate a lognormal distribution.

(5) in another way, it causes the same percent of the variance in the dependent variable to be explained with one less independent variable.

The next step involved the introduction of cross products, and the first selected for examination were those which a priori analysis suggested might exist between years since graduation on one hand and expenditure per pupil, extracurricular activities, and ability on the other. With its significant linear segments, years since graduation has become not one but three variables, and this involved introducing cross products in triplets. All three expenditure per pupil cross products had coefficients insignificantly different from 0; in fact, all three t statistics were less than 1.0. As for extracurricular activities, this variable was so insignificant ($t = .30$) when added to regression 1, Table 2, that an alternative formulation was attempted. Instead of introducing cross products which applied to the whole pilot sample, they were applied to the business sector only, with the idea in mind that the complex of social and cognitive abilities represented by extracurricular activities would be of particular value in the business world. As can be seen from regression 2, Table 3, only $D_4(Y - 36)$ was significant. This variable retained its significant coefficient when the four insignificant variables in regression 2 were removed; in fact, the t statistic rose from 2.25 to 2.85. It states that extracurricular activities are significantly related to income for businessmen who graduated from college more than 36 years prior to 1947.

An F test determined that the cross products involving ability were significant as a group, even though some had insignificant t statistics. As can be seen in regression 3, Table 3, these ability cross products also caused the t statistics of some of the simple variables to drop to insignificance. The question of which insignificant variables to drop was decided by choosing that formulation which contained only significant variables and which, when compared to regression 3, involved the least loss of power to explain variance of the dependent variable. Regression 4, Table 4, met these criteria, and thus it is seen that the cross products were retained but three of the simple variables were eliminated.

TABLE 2

Pilot Study Regression

Independent Variable	Notation	Regression Coefficient	t	R^2
Regression 1				
Constant term		7.307	49.3	.349
Years since graduation	Y	.0375	12.42	
- - -	$D_3(Y - 26)$	-.0794	-6.17	
- - -	$D_4(Y - 36)$	.0579	2.31	
Self-employed	K	.2778	3.89	
Ability	Q	.00745	6.04	
Government	H	-.2436	-3.54	
Education	U	-.3811	-6.81	
Medicine	M_d	.1617	2.15	
Ability Regressions				
(1) Unadjusted grades		.1749	3.18	.319
(2) Adjusted grades	A	.1937	4.64	.330
(3) Mean test score	$\bar{T}$	.00733	5.04	.346
(3) - - -	$(S_T / S_A)(A - \bar{A})$	.00408	3.95	.346
(4) Mean test score	$\bar{T}$	.00613	4.21	.347
(4) Adjusted grades	A	.1744	4.21	.347
(5) "Ability"	Q	.00760	6.14	.346

TABLE 3

Pilot Study Regressions (cont.)

Independent Variable	Notation	Regression Coefficient	t	R^2
Regression 2				
Constant term	--	7.325	47.7	.355
Years since graduation	Y	.0355	9.58	
- - -	$D_3(Y - 26)$	-.0819	-5.20	
- - -	$D_4(Y - 36)$	.0704	2.11	
Self-employed	K	.2494	3.54	
Ability	Q	.00774	6.32	
Government	H	-.2682	-3.68	
Education	U	-.4170	-6.38	
Extracurricular	X	.00161	0.45	
(Business)(Extracurricular)	BX	-.00835	-1.34	
- - -	BXY	.00050	1.28	
- - -	$BXD_3(Y - 26)$	-.00179	-1.07	
- - -	$BXD_4(Y - 36)$	.01287	2.25	
Regression 3				
Constant term	--	8.146	28.4	.356
Years since graduation	Y	-.0276	-1.36	
- - -	$D_3(Y - 26)$	.0690	0.72	
- - -	$D_4(Y - 36)$	-.1643	-0.85	
Self-employed	K	.2751	3.88	
Ability	Q	.00023	0.09	
Government	H	-.2619	-3.86	
Education	U	-.4089	-7.41	
(Ability)(Years since graduation	QY	.00057	3.27	
- - -	$QD_3(Y - 26)$	-.00130	-1.65	
- - -	$QD_4(Y - 36)$	.00185	1.20	

TABLE 4

Pilot Study Regression (cont.)

Independent Variable	Notation	Regression Coefficient	t	R^2
Regression 4				
Constant term	--	8.171	183.8	--
Years since graduation	Y	-.0254	-2.94	
Self-employed	K	.2700	3.84	
Government	H	-.2646	-3.92	
Education	U	-.4112	-7.59	
(Ability)(Years since graduation)	QY	.00055	7.76	
- - -	$QD_3(Y - 26)$	-.00075	-7.15	
- - -	$QD_4(Y - 36)$	.00055	2.82	
Regression 5				
Constant term	--	7.515	45.6	.411
Years since graduation	Y	-.0194	-2.11	
Self-employed	K	.2859	4.10	
(Ability)(Years since graduation)	QY	.00052	6.88	
- - -	$QD_3(Y - 26)$	-.00082	-7.45	
- - -	$QD_4(Y - 36)$	.00086	3.58	
(Medicine)(Years since graduation)	M_dY	.0215	4.76	
- - -	$M_dD_4(Y - 36)$	-.1932	-3.80	
(Medicine)(City size)	M_dOZ	-.00055	-2.24	
(Government)(Years since graduation)	HY	-.0118	-2.93	
(Education)(Years since graduation)	UY	-.0169	-5.28	
- - -	$BXD_4(Y - 36)$	-.0095	2.83	
(Business)(City size)	BOZ	.00022	2.22	
(Business)(Mother's education)	BM	.3093	2.90	
(Business)(Father's education)	BF	.1204	1.75	
Expenditure per pupil	E	.000047	1.00	
Enrollment	log (N)	.0556	3.25	

TABLE 5

Pilot Study Regression (cont.)

Independent Variable	Notation	Regression Coefficient	t
Regression 6			
Constant term	--	8.022	180.6
Years since graduation	Y	-.0236	-2.71
Self-employed	K	.2866	4.14
(Ability)(Years since graduation)	QY	.00056	7.80
- - -	$QD_3(Y - 26)$	-.00081	-7.29
- - -	$QD_4(Y - 36)$	.00089	3.71
(Medicine)(Years since graduation)	M_dY	.0181	3.69
- - -	$M_dD_4(Y - 36)$	-.1740	-3.33
(Medicine)(City size)	M_dOZ	-.00060	-2.40
(Government)(Years since graduation)	HY	-.0116	-2.91
(Education)(Years since graduation)	UY	-.0190	-5.75
- - -	$BXD_4(Y - 36)$	.0083	2.47
(Business)(City size)	BOZ	.00023	2.35
(Business)(Mother's education)	BM	.3178	2.97
(Business)(Father's education)	BF	.1419	2.07
Years of graduate education	J	.00656	0.18
- - -	D(J - 1.5)*	.0380	0.64
Final Test Regressions			
Catholic college	C	-.0946	-1.25
Veteran	V	-.0405	-0.86
Region — South	S	.0268	0.48
Region — West	W	-.0075	-0.13
Both parents attended college	P	.0699	0.89
Extracurricular	X	.00107	0.41
College self-support	S_p	.000088	1.44
Medicine	M_d	.2190	1.31
Law	L	.0639	0.69
Government	H	.0441	0.35
Education	U	-.1167	-1.20

*D = 1 if J > 1.5, otherwise D = 0.

The next group of cross products to be examined were those between occupation and years since graduation. Since there were four occupation variables (business was the omitted category), and three years-since-graduation variables, this meant the introduction of twelve cross products. All four simple occupation variables were also introduced. The insignificant variables were then progressively eliminated, those with t statistics closest to 0 being eliminated first, until only significant variables remained.

The only other cross products examined were those between occupation on the one hand and city size, parents' education, and region on the other. All possible combinations of occupation-city size and occupation-parents' education were tried; the city size cross products (where city size is OZ in the notation presented above) were significant only for business and medicine, while the only significant parents' education cross products were those for business. The only regional cross product examined was that for medicine, since incomes in this occupation are presumed to be most directly related to general regional wage levels.[1] When the coefficients for these cross products were seen to be insignificant, the occupation-region cross products were carried no further.

This completed the search for determinants of income which should be accounted for when the relationship between education and income is examined. The final equations for the two education variables considered, expenditure per pupil and years of graduate education, are shown as regression 5, Table 4, and regression 6, Table 5.

A final test, however, was given to the simple variables excluded from the final equations. They were reintroduced one at a time as one final check to make sure that their coefficients differed insignificantly from 0. The coefficients and t statistics of these variables are shown in Table 5 under the title Final Test Regressions. Since none of these final-test variables did in fact have significant coefficients, regressions 5 and 6 were the final equations of the pilot study.

The Main Study

The results of the pilot study permit formulation of a set of hypotheses which may be tested on the remaining, main body of data, 2,625 observations in all. In both regressions 7 and 8 of Table 6, which are repetitions of regressions 5 and 6 of the pilot study, there exists a null hypothesis corresponding to each variable in the regression. The null hypothesis is that the coefficient of the particular variable is insignificantly different from 0 in that particular regression equation. In those cases where this null hypothesis must be rejected, with the probability of type I error of .05,[2] the alternate hypothesis of relationship between the dependent variable and the particular independent variable is

1. Friedman and Kuznets, op. cit., pp. 195–96; 229–35.

2. This is the probability that the hypothesis being tested will be rejected when it is true. A two-tailed test is used.

TABLE 6

Main Study Regressions

Independent Variable	Notation	Regression Coefficient	t	R^2
Regression 7				
Constant term	--	7.691	90.0	.362
Years since graduation	Y	.0013	0.25	
Self-employed	K	.2470	6.31	
(Ability)(Years since graduation)	QY	.00037	8.69	
- - -	$QD_3(Y - 26)$	-.00063	-11.13	
- - -	$QD_4(Y - 36)$	.00017	1.21	
(Medicine)(Years since graduation)	M_dY	.0201	8.50	
- - -	$M_dD_4(Y - 36)$	-.0287	-1.26	
(Medicine)(City size)	M_dOZ	-.00019	-1.27	
(Government)(Years since graduation)	HY	-.0129	-6.07	
(Education)(Years since graduation)	UY	-.0182	-10.68	
- - -	$BXD_4(Y - 36)$	-.0006	-0.25	
(Business)(City size)	BOZ	.00011	2.15	
(Business)(Mother's education)	BM	.1588	2.81	
(Business)(Father's education)	BF	.0527	1.40	
Expenditure per pupil	E	.000039	1.67	
Enrollment	log (N)	.0339	3.80	
Regression 8				
Constant term	--	8.001	334.0	.360
Years since graduation	Y	-.0026	-0.53	
Self-employed	K	.2525	6.44	
(Ability)(Years since graduation)	QY	.00040	10.07	
- - -	$QD_3(Y - 26)$	-.00062	-10.90	
- - -	$QD_4(Y - 36)$	.00015	1.11	
(Medicine)(Years since graduation)	M_dY	.0174	6.61	
- - -	$M_dD_4(Y - 36)$	-.0173	-0.74	
(Medicine)(City size)	M_dOZ	-.00024	-1.61	
(Government)(Years since graduation)	HY	-.0131	-6.16	
(Education)(Years since graduation)	UY	-.0199	-11.29	
- - -	$BXD_4(Y - 36)$	-.0008	-0.33	
(Business)(City size)	BOZ	.00013	2.41	
(Business)(Mother's Education)	BM	.1681	2.97	
(Business)(Father's Education)	BF	.0642	1.70	
Years of graduate education	J	.0132	0.67	
- - -	D(J - 1.5)	.0221	0.66	

accepted. Furthermore, by the a priori reasoning of the previous sections, it is concluded that the particular independent variable is either a determinant or is a proxy for one or more determinants of earned income.

From regressions 7 and 8 we therefore conclude that the following variables are in some additive or multiplicative way determinants of earned income: ability, occupation, years since graduation, whether or not self-employed (K), size of college (log N), and, for businessmen only, city size (BOZ) and mother's education (BM).

It seems intuitively obvious that cognitive ability should be a determinant of income, and yet some studies have reached conclusions which would seem to deny this.[1] Havemann and West found that the Time data showed a relation between grades and income, however, and this reworking of the data does not dispute their conclusion. The variables involving occupation, i.e., medicine, government, and education, are significant as compared to the omitted category, which is a combined category of business and law. In regression 7 the significant positive coefficient of the cross product of medicine and years since graduation (M_dY) shows that incomes in medicine are significantly greater than in the other occupations even when ability differences are considered. In regression 8, length of graduate training has been introduced, and the income advantage of medicine, measured by the M_dY coefficient, has declined somewhat, but it still remains substantial.

The occupation variable, it will be remembered, can be a proxy for quasi-rents, differences in nonmoney income, or restrictionism. In view of the great status accorded members of the medical profession it seems difficult to argue that nonmoney income is so low in medicine that the significant regression coefficient represents a compensating differential. Furthermore, this differential has been so long-lived—it was, for example, present in the Friedman and Kuznets study which used data from the early 1930's—that it cannot be called a quasi-rent as that term is usually used. It seems most likely, therefore, that at least part and probably all of the explanation must lie with restrictions on supply.

This is not to say, however, that the medical profession deliberately restricts entry in order to raise medical income. Any intentional entry restrictions could be undertaken solely for the purpose of improving standards of practice, with increased medical incomes an unintended by-product. Alternatively, it could be that supply is unresponsive because the producing firms, i.e., the medical schools, are not profit maximizing institutions and are not influenced by the market forces which guide production in other industries. A regression analysis such as this cannot indicate the relative importance of these three possible causes of supply restrictions.

---

1. Gambrill, op. cit.; Victor Jepsen, "Scholastic Proficiency and Vocational Success," Educational and Psychological Measurement, Vol. 11 (Winter 1951), pp. 616–28.

The significantly lower incomes in government and education may be attributed in large part to nonmoney factors such as security, interesting work, and social purpose. These differentials may have been particularly large in 1947, however, as money income paid by governments took time to adjust to new postwar price levels. In other words, other occupations may have been receiving quasi-rents at the expense of civil servants and teachers.[1]

Years since graduation (Y) is quite evidently a determinant of income through its close relationship with work experience. As was mentioned above, the self-employment variable (K) can represent a premium for risk, but it is not certain whether this premium should be positive or negative. In view of the magnitude of the positive differential associated with self-employment, however, it seems reasonable to suspect that the variable represents at least in part the returns to capital rather than labor.

The size of college variable (log N) acquired an unforeseen importance. Introduced only to handle the problem of overhead costs in expenditure per pupil calculations, it evidently stands for much more than this, since it is significant while expenditure per pupil is not. It is a proxy for some combination of other determinants, but the nature of these determinants is so obscure that the variable confuses more than it clarifies. By a process of ex post rationalizing, however, the following determinants may be suggested. The variable may represent market imperfection to the extent that graduates of large colleges face a better organized, nationwide job market. Or it may be a proxy for motivation as those more inclined to spiritual activities and less inclined to economic achievement may tend to attend small, religiously oriented colleges. Or, finally, it may be a proxy for those determinants which are presumed to lie behind the city size variable. Note, for example, that when the size of college variable is present in regression 7, the business-city size cross product (BOZ) is lower in both magnitude and significance than it is in regression 8.

The reasons are equally obscure for explaining why city size is a determinant of income in business but not in other occupations. The answers could be revealed only in a more detailed examination than is possible in the present study.[2] As for the final significant variable, mother's education (BM),

1. In the case of teachers, their income declined relative to the general wage level in the immediate postwar years, but by 1947 they had virtually regained their relative position of 1929. See Committee for Economic Development, Paying for Better Schools (New York: CED, 1959), pp. 16–18.

2. The insignificance of the city size variable among all occupations except business is a result which contrasts with the results of Friedman and Kuznets (op. cit., p. 184), who found incomes to be much lower in small cities. Evidently low incomes in small communities can be explained by other factors, possibly ability. The negative slope of the city size variable for doctors, although not significant, is in agreement with their finding that doctors in large cities earn low incomes.

this will be discussed more fully in the following section. It may be noted here, however, that it will be argued that this is primarily a measure of motivation rather than social class, and therefore no significant social class variable emerges from this regression analysis. The market imperfections which were supposed to hinge on social class do not seem to be of great importance, and these results therefore suggest that there is substantial equality of economic opportunity among college graduates.[1]

This is not to say, however, that social class has no bearing on income for college graduates. The ability variable represents the educational advantages of wealth and home environment as well as the individual's endowment from heredity. The democratic ideal requires equal opportunity to prepare for work as well as equal opportunity in the job market itself.

6. Other Regression Tests

In this study, the primary function of the main body of data is to show the relation between education and income when other variables, deemed significant by the pilot study, are controlled. However, this body of data was also used to examine other hypotheses regarding the determinants of earned income.

Self-Support in College

In analyzing the Time data, Havemann and West noted that students who worked their way through college earned lower incomes in later life than those who were supported by others during their college careers. They explained this by suggesting that:

> "What all this amounts to is that the student who works his way starts his career under an economic handicap which, although hardly burdensome at all at the beginning, weighs more and more heavily as the race goes on. . . . Perhaps part of the explanation is that family assistance for the supported student does not necessarily end on cap-and-gown day. Thanks to his background or family influence he may start out in jobs with a better chance for advancement."[2]

This disadvantageous position of the self-supporting student is shown in regression 9 of Table 7. The negative coefficient of self-support is significant, and says that as the percent of college self-support increases, income in later life declines. Regression 10 of Table 7, however, shows an entirely different picture. Now the college support variable has been added to a regression containing all the variables deemed significant by the pilot

1. A possible exception to this statement lies in the fact that occupation may become a proxy for social class if medical schools use this as one of their criteria for admission.

2. Havemann and West, op. cit., pp. 171–72.

study, i.e., all the variables in regressions 7 and 8, except those pertaining to education. The t statistic of the college support coefficient provides a test of the null hypothesis that, when other factors are controlled, college self-support is not significantly related to income. The null hypothesis cannot be rejected, and in fact the sign of the coefficient has been reversed. The inference to be derived from regression 10, therefore, is that the person who worked his way through college seems to have a slight advantage, not a disadvantage, in subsequent employment, but his advantage is not statistically significant.

Princeton versus Podunk

Havemann and West also examined income differences between graduates from colleges of different quality. They found that "earning power rises steadily with each increase in wealth and prestige of the school," and that "the figures we have been examining here are quite a testimonial to the rich and famous schools, and especially to the Ivy League. It is worth asking whether the figures are perhaps even more of a testimonial to the families of the boys who go there." They conclude that "the golden touch of the Ivy League falls on rich and poor alike."[1] In other words, the authors suggest that prestigious colleges bestow a blessing on their graduates, an economic advantage which is at least in part independent of the graduates' actual ability. Although the proposition is not explicitly stated, the implication of this reasoning is that any individual of given ability will achieve higher income in life if he goes to a better college. This proposition may be tested by introducing into the regression equation a proxy variable for a college's prestige. The proxy chosen depends on what one considers to be the source of prestige; the case for prestige being based on the mean ability level of the student body appears at least as strong as any alternative case that might be made, and so this variable appears in regression 11, Table 8, along with the variables found significant in the pilot study. Although the coefficient for mean test score is positive, it is not sufficiently large to permit rejection of the null hypothesis that there is no relation between an individual's income and this measure of his college's prestige. We must conclude, therefore, that although this prestige factor may be an income determinant, the Time data do not conclusively demonstrate its existence, and most if not all of the income differences which Havemann and West attributed to prestige are more properly assigned to ability. The C student from Princeton earns more than the A student from Podunk not mainly because he has the prestige of a Princeton degree, but merely because he is abler. The golden touch is possessed not by the Ivy League college, but by its students.[2]

1. Ibid., pp. 180–81.

2. For an expansion of this idea into the field of values and personality traits, see T. R. McConnell and Paul Heist, "Do Students Make the College?" College and University, Vol. 34 (Summer 1959), pp. 442–52.

TABLE 7

Main Study Regressions

Special Tests

Independent Variable	Notation	Regression Coefficient	t	R^2
Regression 9				
Constant term	--	8.065	279.81	.223
Years since graduation	Y	.0428	25.11	
- - -	$D_3(Y - 26)$	-.0693	-9.41	
- - -	$D_4(Y - 36)$	.0338	2.05	
College self-support	S_p	-.00095	-2.72	
Regression 10				
Constant term	--	7.992	290.0	.359
Years since graduation	Y	-.0042	-1.87	
Self-employed	K	.2513	6.41	
(Ability)(Years since graduation)	QY	.00042	10.51	
- - -	$QD_3(Y - 26)$	-.00064	-11.31	
- - -	$QD_4(Y - 36)$	.00018	1.30	
(Medicine)(Years since graduation)	M_dY	.0205	8.65	
- - -	$M_dD_4(Y - 36)$	-.0297	-1.30	
(Medicine)(City size)	M_dOZ	-.00018	-1.19	
(Government)(Years since graduation)	HY	-.0131	-6.16	
(Education)(Years since graduation)	UY	-.0193	-11.33	
- - -	$BXD_4(Y - 36)$	-.0007	-0.28	
(Business)(City size)	BOZ	.00011	2.19	
(Business)(Mother's education)	BM	.1625	2.87	
(Business)(Father's education)	BF	.0634	1.68	
College self-support	S_p	.00044	1.35	

Extracurricular Activities

The education literature contains several studies which attempt to assess the value of college grade averages and extracurricular activity records for predicting future income.[1] The results are by no means uniform; Gambrill and Jepsen found extracurricular activities to be the more important predictor, for example, while Bridgman and Havemann and West concluded that, unlike grades, extracurricular activities had no predictive value at all. Even though the pilot study found virtually no relation between extracurricular activities and income—the t statistic was 0.41—the conflicting nature of previous results suggested that it would be useful to examine the relation between these two variables in the main body of data. This was done in regression 12, Table 8; the results were surprising, for the extracurricular activity variable is seen to be significantly related to income, with a t statistic of 4.0. Thus it turned out that in this particular characteristic the sample drawn for the pilot study was completely unrepresentative of the rest of the data. This is also shown by the drastic change in the t statistic for the $BXD_4(Y - 36)$ variable, which shifted from 2.4 or higher in the pilot study to around −0.3 in the main data. The conclusion to be derived from the Time data, therefore, is that both grades and extracurricular activities are related to income. The relative importance of these two variables cannot be assessed from regression 12 since grades are not a separate variable.

The Education of Parents

In the pilot study the relationship between parents' education and income seemed particularly curious. First, the pilot study showed a significant relationship only among those graduates who went into business. Second, among businessmen the relationship was particularly significant if the mother but not the father attended college. Parents' education, it may be remembered, was represented by a set of dummy variables indicating which parent attended college, the omitted category being that neither parent attended. The t statistics of the three variables were: only father attended (BF), 1.9; only mother attended (BM), 3.0; both parents attended (BP), 0.9.

If parents' education is truly a proxy for social class, the relative significance of these three variables would be expected to be the exact opposite of what the pilot study actually showed. The main body of data was therefore used to test the relationships between these three variables and earned income. The results, shown in regressions 13 and 14 of Table 9, are virtually the same as in the pilot study. In regression 13 the parents' education variables apply to businessmen only, and the t statistics differ very little from what they were in the pilot study. BF is 1.7, BM is 2.9,

1. Gambrill, op. cit.; Bridgman, op. cit.; Jepsen, op. cit.; Havemann and West, op. cit., Chapter 13.

TABLE 8

Main Study Regressions

Special Tests (cont.)

Independent Variable	Notation	Regression Coefficient	t	R^2
Regression 11				
Constant term	--	7.890	66.17	.359
Years since graduation	Y	.0003	0.06	
Self-employed	K	.2490	6.34	
(Ability)(Years since graduation)	QY	.00038	7.09	
- - -	$QD_3(Y - 26)$	-.00063	-11.12	
- - -	$QD_4(Y - 36)$	.00018	1.33	
(Medicine)(Years since graduation)	M_dY	.0206	8.66	
- - -	$M_dD_4(Y - 36)$	-.0296	-1.30	
(Medicine)(City size)	M_dOZ	-.00019	-1.26	
(Government)(Years since graduation)	HY	-.0130	-6.09	
(Education)(Years since graduation)	UY	-.0190	-11.21	
- - -	$BXD_4(Y - 36)$	-.0007	-0.28	
(Business)(City size)	BOZ	.00011	2.17	
(Business)(Mother's education)	BM	.1590	2.81	
(Business)(Father's education)	BF	.0581	1.54	
Mean test score	$\bar{T}_p$	.0011	1.04	
Regression 12				
Constant term	--	7.959	296.7	.363
Years since graduation	Y	-.0048	-0.99	
Self-employed	K	.2471	6.32	
(Ability)(Years since graduation)	QY	.00043	10.74	
- - -	$QD_3(Y - 26)$	-.00064	-11.46	
- - -	$QD_4(Y - 36)$	.00021	1.51	
(Medicine)(Years since graduation)	M_dY	.0208	8.80	
- - -	$M_dD_4(Y - 36)$	-.0332	-1.46	
(Medicine)(City size)	M_dOZ	-.00018	-1.20	
(Government)(Years since graduation)	HY	-.0130	-6.10	
(Education)(Years since graduation)	UY	-.0197	-11.61	
- - -	$BXD_4(Y - 36)$	-.0012	-0.50	
(Business)(City size)	BOZ	.00011	2.10	
(Business)(Mother's education)	BM	.1539	2.73	
(Business)(Father's education)	BF	.0604	1.61	
Extracurricular	X	.0052	4.01	

and BP is 1.2. Therefore the null hypotheses of no relationship between income on one hand and BF and BP on the other cannot be rejected, but the hypothesis of no relation between income and BM can be rejected at the 1% level of significance.

Regression 14 examines the hypothesis of relation between parents' education and income in occupations other than business. Since the "both parents attended" variable had made such a poor showing in all previous regressions, it was not included here, so only F and M variables were introduced. Their t statistics proved to be 0.86 and 0.29 respectively, and so the null hypotheses of no relationship with income were not rejected.

How may these results be explained? What advantage in the job market can be gained by the person whose mother attended college but whose father didn't? It does not seem likely that this is an advantage of family influence, or status, because these factors are largely controlled by the father's occupation and would therefore be more closely associated with the father's education. Nor does it seem likely that the explanation is that the individual came for a poor but able family and therefore had to work his way through college, neglecting his grades in the process. This argument would suggest collinearity between college self-support and BM, but a comparison of regression 10, Table 7, with regressions 7 and 8, Table 6, shows that collinearity is not present, since the BM coefficient is not affected by introducing the college self-support variable into the regression. The only explanation which can be offered, therefore, is that the "mother only" variable is a proxy for motivation. This could be the case if values and motivation acquired from the home environment come largely from the mother, and a woman who has married beneath her educational level becomes particularly anxious to motivate her children to the status and achievement which she has denied herself. For more intensive research on this point, however, the economist must defer to members of other social sciences.[1]

Race

A further explanation is in order as to why an apparently obvious determinant of income, race, has not been mentioned up till now. The reason is that there were only 22 Negroes in the data, 3 in the pilot study, and 19 in the main data. Regression 15, Table 10, contains a race variable, coded 0 for Negroes and 1 for whites, and it is seen that these 19 Negroes earned

1. Another study reporting a similar statistical result may be noted. In studying the determinants of education levels for adult offspring in rural Tennessee, Hughes found that the variable measuring educational attainment of the mother was more significant ($t = 6.0$) than that of the father ($t = 2.4$). See Rufus B. Hughes, "Low Incomes in Southern Agriculture Population Adjustments" (unpublished Ph. D. dissertation, University of Chicago, 1959), p. 13.

TABLE 9

Main Study Regressions

Special Tests (cont.)

Independent Variable	Notation	Regression Coefficient	t	R^2
Regression 13				
Constant term	--	8.001	333.6	.359
Years since graduation	Y	-.0035	-0.73	
Self-employed	K	.2495	6.36	
(Ability)(Years since graduation)	QY	.00042	10.40	
- - -	$QD_3(Y - 26)$	-.00064	-11.36	
- - -	$QD_4(Y - 36)$	.00018	1.33	
(Medicine)(Years since graduation)	M_dY	.0206	8.69	
- - -	$M_dD_4(Y - 36)$	-.0299	-1.31	
(Medicine)(City size)	M_dOZ	-.00018	-1.22	
(Government)(Years since graduation)	HY	-.0128	-5.97	
(Education)(Years since graduation)	UY	-.0188	-11.02	
- - -	$BXD_4(Y - 36)$	-.0006	-0.25	
(Business)(City size)	BOZ	.00011	2.15	
(Business)(Mother's education)	BM	.1654	2.91	
(Business)(Father's education)	BF	.0663	1.75	
(Business)(Both parents attended college)	BP	.0498	1.23	
Regression 14				
Constant term	--	8.008	336.0	.359
Years since graduation	Y	-.0035	-0.71	
Self-employed	K	.2513	6.40	
(Ability)(Years since graduation)	QY	.00041	10.38	
- - -	$QD_3(Y - 26)$	-.00064	-11.31	
- - -	$QD_4(Y - 36)$	.00018	1.29	
(Medicine)(Years since graduation)	M_dY	.0201	8.37	
- - -	$M_dD_4(Y - 36)$	-.0282	-1.24	
(Medicine)(City size)	M_dOZ	-.00018	-1.25	
(Government)(Years since graduation)	HY	-.0133	-6.17	
(Education)(Years since graduation)	UY	-.0192	-11.23	
- - -	$BXD_4(Y - 36)$	-.0007	-0.26	
(Business)(City size)	BOZ	.00012	2.24	
(Business)(Mother's education)	BM	.1425	1.70	
(Business)(Father's education)	BF	.0241	0.42	
Father's education	F	.0381	0.86	
Mother's education	M	.0185	0.29	

lower incomes than whites who were similar in other characteristics, but that the difference was not statistically significant. A sample of 19, however, is of little use for making generalizations applicable to all of society. The most likely reason for having such a small number of Negroes in such a large sample of college graduates is that Negro colleges tend to be too poor and understaffed to do a good job of keeping track of alumni.

7. Rates of Return on Investment in Education

In order to compute rates of return, it is necessary to assign mean values of income to each particular combination of education level and years since graduation. These mean income values may be derived in any of three different ways. First, each observation from the main body of data may be assigned to its education years-since-graduation cell, and the mean income computed for all observations in each cell. This is the approach

TABLE 10

Main Study Regressions

Special Tests (cont.)

Independent Variable	Notation	Regression Coefficient	t	R^2
Regression 15				
Constant term	- -	7.818	63.39	.359
Years since graduation	Y	-.0029	-0.59	
Self-employed	K	.2490	6.35	
(Ability)(Years since graduation)	QY	.00041	10.18	
- - -	$QD_3(Y - 26)$	-.00064	-11.32	
- - -	$QD_4(Y - 36)$	.00018	1.32	
(Medicine)(Years since graduation)	M_dY	.0205	8.67	
- - -	$M_dD_4(Y - 36)$	-.0297	-1.31	
(Medicine)(City size)	M_dOZ	-.00019	-1.26	
(Government)(Years since graduation)	HY	-.0130	-6.11	
(Education)(Years since graduation)	UY	-.0190	-11.23	
- - -	$BXD_4(Y - 36)$	-.0007	-0.27	
(Business)(City size)	BOZ	.00012	2.20	
(Business)(Mother's education)	BM	.1604	2.84	
(Business)(Father's education)	BF	.0615	1.63	
Race	R	.1949	1.60	

used by previous studies in this field. No account is taken of income determinants other than education and years since graduation.

This same result can be obtained with a regression equation which expresses income as a function of these two determinants alone. Regressions 16 and 17 of Table 11 are two such equations. An income mean for any particular cell may be computed by inserting the education and years-since-graduation values which define the cell into the equations:

$$\log I = 7.924 + .0423\,Y - .0701\,D_3(Y - 26) + .0363\,D_4(Y - 36) + .0001083\,E$$

$$\log I = 8.025 + .0428\,Y - .0693\,D_3(Y - 26) + .0358\,D_4(Y - 36) - .0877\,J + .2227\,D(J - 1.5)$$

and solving for I. The rates of return computed from income means derived in these two ways should be the same.

The third method to be used computes rates of return that control for the influence of other income determinants. This involves using partial regression equations, in which variables involving education and years since graduation are assigned the coefficients they acquired in regressions 7 and 8 of Table 6. All other variables in these two regressions are incorporated into new constant terms, which equal the old constant terms plus the sum of

TABLE 11

Regression of Income on Education Level and Years since Graduation

Independent Variable	Notation	Regression Coefficient	t	R^2
Regression 16				
Constant term	--	7.924	239.2	.226
Years since graduation	Y	.0423	24.82	
- - -	$D_3(Y - 26)$	-.0701	-9.54	
- - -	$D_4(Y - 36)$	.0363	2.20	
Expenditure per pupil	E	.0001083	4.45	
Regression 17				
Constant term	—	8.025	327.0	--
Years since graduation	Y	.0428	24.85	
- - -	$D_3(Y - 26)$	-.0693	-9.44	
- - -	$D_4(Y - 36)$	.0358	2.19	
Years of graduate education	J	-.0877	-4.32	
- - -	D(J - 1.5)	.2227	6.54	

the coefficients times the mean values of all the variables not involving education or years since graduation.[1] Since each coefficient may be considered a maximum likelihood estimate, it is retained in the partial regression equation, whether as an independent variable or as a component of the constant term, even in cases where the t statistic shows it to be insignificantly different from zero.

It will be seen that the years-since-graduation variable appears in regressions 7 and 8 in cross products with ability, occupation, and extracurricular activities. The income means for given education/years-since-graduation cells will therefore depend on what values are assigned to these three other variables. However, experimentation with various values of extracurricular activities showed that this variable had negligible influence on income means at the level of accuracy used in these computations, so in effect there will exist a set of income means for every particular combination of occupation and ability level.

Expenditure per Pupil

We turn first to measurements of the rate of return on investment in additional resources to improve the quality of undergraduate education, and the relevant educational measure is taken to be expenditure per pupil. By this measure two classes of educational quality were defined, the dividing line between the two being a 1953–54 expenditure per pupil of $1,000. Forty-four percent of the observations in the main body of data were above this dividing line, and had an average expenditure of $1,511; 56% were below with a mean expenditure of $696.

It is assumed that individuals retire fifty years after they enter college, the time at which the decision on college quality must be made. This fifty-year span is divided into ten five-year classes, and it is assumed, with one exception, that all income is received in the middle year of the five-year span.[2] The one exception is the first of the five-year spans. The first four years of this period are spent in college, so it is only in the last of these five years that an income differential appears between graduates of high and of low expenditure colleges.[3] This differential is assigned directly to this first year after graduation, and the rate of return is therefore that value r such that:

---

1. See M. Ezekiel and K. Fox, Methods of Correlation and Regression Analysis (3d ed.; New York: John Wiley, 1959), Chapter 14.

2. If the same income is received in each of the five years, this assumption will cause the present value of the income from one such span to be overstated by 1% if the rate of return is .10, by 3% if the rate of return is .20.

3. It is assumed, therefore, that there is no difference in the income of students in colleges of different expenditure level while they are actually in college.

$$\sum_{i=0}^{3} M_i(.83)(1511 - 696)\frac{1}{(1+r)^i} = M_4\,\Delta I_4\frac{1}{(1+r)^4} + \sum_{i=1}^{9} M_{5i+2}\,(5\,\Delta I_{5i+2})\frac{1}{(1+r)^{5i+2}}$$

M_i is a mortality factor based on 1947 mortality experience for the whole white male population.[1] It is defined as the percent of 18 year olds who will survive to age 18 + j, where j is the subscript of M. ΔI_i is the income differential, i years after entering college, between graduates of high and of low expenditure colleges. Expenditure figures are multiplied by the factor 0.83 to convert 1953–54 values to the 1947 level, making them comparable with 1947 income figures.[2]

The results, computed to the nearest 0.2%, show that the expenditure per pupil rate of return is 12.0% when other determinants are not considered. This same result is obtained no matter which way the computations are undertaken, whether by assigning observations to cells or by using the coefficients of regression 16, Table 11. As can be seen from Table 12, however, the results are quite different when other determinants are controlled through the use of coefficients from a partial regression equation. Now the computed returns vary with occupation and ability level, but a representative return would lie between 5 and 6 percent. We may therefore conclude that when other measurable determinants of income are taken into account the rate of return on this type of investment in education is cut by about 50%.[3]

---

1. National Office of Vital Statistics, U. S. Public Health Service, Vital Statistics of the United States, 1947, Part 1 (Washington: U. S. Government Printing Office, 1949), p. xlvii.

2. The U. S. Office of Education reports an aggregate expenditure per pupil in colleges of $409 in 1953–54, and $341 in 1947–48. 341/409 = 0.83. These figures are low compared to others used in this study mainly because interest and depreciation on plant and equipment are not considered. (U. S. Office of Education, Biennial Survey of Education in the United States, 1954–56 (Washington: U. S. Government Printing Office, 1959), Chapter 4, Section 1, p. 8; Section 2, p. 7.)

3. The regression form used is not well suited for measuring rate of return differences among various occupations or ability levels, since there are no cross products between occupation and ability on one hand and the education variables on the other hand. The differences shown in Table 12 and 15 result entirely from the choice of a logarithmic form for the dependent variable. Since 60% of the sample is included in business, the business rates of return provide the best estimate of overall controlled rates of return.

TABLE 12

Rates of Return—Expenditure per Pupil

	Ability Level*				
	150	125	100	75	50
Business	.064	.056	.050	.042	.036
Medicine	.080**				
Education					.018***

*The scale used for measuring ability is based on raw scores of the 1952 edition, American Council on Education Psychological Examination. These are not IQ scores. For all freshmen entering college in 1952, the mean score was 105 with a standard deviation of 27 (cf. Darley, et al., op. cit., Chapter 2).

**Highest computed rate.

***Lowest computed rate.

Graduate Education

Three educational classes were chosen for the analysis of rates of return on investment in graduate (and professional) education. There is, first, 0 years beyond the bachelor's degree, the standard to which other levels will be compared. The second class is 1 1/2 years, which consists largely of people with master's degrees, but which also includes a few doctors whose first and only degree was an M. D. received between 1913 and 1920. The third class is 3 or 3 1/2 years, which includes all people with non-medical doctor's degrees, initial M. D.'s received after 1937, B. D.'s, or LL. B.'s granted after the person received an ordinary bachelor's degree. All individuals in this group will be considered to have 3 1/4 years of graduate training. Unlike the educational classes for expenditure per pupil, the present classes are not all-inclusive. A person with, say, 2 years of graduate training is excluded from the computations.

The principal change which must be made in the years-since-graduation classes is that now both the first and second of the five-year classes must have separate treatment for each year. This is because the graduate education of classes 2 and 3 spills over into the second five-year class.

The measurement of costs becomes more difficult than before for two reasons. First, the direct costs of education are the expenditures devoted to each graduate student's education, but the expenditure per pupil figures which have been collected pertain to undergraduate education. Moreover, the relationship between per pupil costs at these two educational levels is very imperfectly known. The few scraps of information available suggest a graduate-

undergraduate expenditure per pupil ratio of 2.0, but this is such a vague estimate that rates of return are also worked out with alternative ratios of 1.5 and 3.0.[1] Second, a new form of cost enters the picture: foregone earnings. To estimate these earnings requires an estimate of the income of students while they are in graduate schools.

The usual assumption which as been made in the case of undergraduates is that they are employed only during the summer vacation, when they earn incomes similar to those who did not go to college. This amounts to an estimate that students earn 20 or 25 percent of the income received by those high school graduates of the same age who are full time workers.[2] This figure seems too high for graduate students for two reasons. First, a higher percent of graduate students may be expected to devote at least part of their summers to academic work, and second, those who do work are not likely to receive weekly incomes as high as the college graduate who is engaged full time in a career job. Accordingly the assumption used for the present analysis is that graduate students earn 15% of the income received by those of the same age who did not go on to graduate training.

The rate of return on investment in 3 1/4 years of graduate education is therefore that rate r such that:

$$0.83R \sum_{i=0}^{2} L_i(1009 - 0)\frac{1}{(1+r)^{4+i}} + (0.25)\,0.83R\; L_3\,(1009 - 0)\frac{1}{(1+r)^{7}}$$

$$= \sum_{i=0}^{5} L_i\,(I''_{4+i} - I_{4+i})\frac{1}{(1+r)^{4+i}}$$

$$+ 5 \sum_{i=1}^{8} L_{5i+3}(I''_{5i+7} - I_{5i+7})\frac{1}{(1+r)^{5i+7}}$$

Now L_j is a new mortality factor which shows the percent of 22 year olds who will survive to age 22 + j, and R is the ratio of graduate to undergraduate expenditure per pupil. The average undergraduate expenditure per pupil in the colleges where these individuals did their 3 1/4 years of graduate work was $1,009. The income of these individuals is designated by I", whereas the income of those with no graduate work is I. Finally, the further constraints exist, by assumption, that:

$$I''_i = (0.15)(I_i), \quad i = 4, 5, 6$$

$$I''_7 = (0.25)(0.15)(I_7) + (0.75)(I^*_7),$$

1. See Hunt, op. cit., Appendix C.

2. Schultz estimates that students forego 40 weeks of income, which would mean that they earn about 20% of the full time worker's annual income. [Theodore W. Schultz, "Capital Formation by Education," Journal of Political Economy, Vol. 68 (December 1960), pp. 573–74.] Becker uses an estimate of 25% in his study.

where I_7^* is the annual income from the regression, or from the years-since-graduation/education cells, remembering that the data pertain only to persons who have completed their formal education.

Similarly the rate of return on investment in 1 1/2 years of graduate education is that the rate r such that:

$$0.83R\, L_0(869 - 0)\frac{1}{(1+r)^4} + (0.5)(0.83)\, L_1\,(869 - 0)\frac{1}{(1+r)^5}$$

$$= \sum_{i=0}^{5} L_i\,(I'_{4+i} - I_{4+i})\frac{1}{(1+r)^{4+i}}$$

$$+ 5\sum_{i=1}^{8} L_{5i+3}(I''_{5i+7} - I_{5i+7})\frac{1}{(1+r)^{5i+7}}$$

The income of individuals with 1 1/2 years of graduate work is denoted by I', and by assumption:

$$I'_4 = (0.15)(I_4)$$

$$I'_5 = (0.5)(0.15)(I_5) + (0.5)(I_5^*).$$

The rates of return computed by these equations without regard for other income determinants produced the results shown in Table 13. These results seem paradoxical for two reasons. First, it seems strange to conceive of education as an investment process if its rate of return is −1.0. This result says that the reward for such investment is lower income in the future. Second, the two methods which were supposed to yield similar results in fact produced drastically different rates of return for investment in 3 1/4 years of graduate training.

TABLE 13

Rates of Return—Graduate Education

	Graduate/Undergraduate Expenditure Ratios		
	1.5	2.0	3.0
1. 3 1/4 years graduate education			
a. by assignment to cells	−1.0	−1.0	−1.0
b. by uncontrolled regression	.030	.024	.022
2. 1 1/2 years graduate education			
a. by assignment to cells	−1.0	−1.0	−1.0
b. by uncontrolled regression	−1.0	−1.0	−1.0

The cause of these different results is apparent from Table 14, showing the mean income values which result from assigning observations to their particular cells and averaging the observations within each cell. In order to have a positive rate of return on investment in 3 1/4 years of graduate education, those who have such education must receive higher incomes in later life to make up for the income losses they sustain in their years as graduate students. This they do during the period from 4 to 18 years after receiving a bachelor's degree, but, if the data could be believed, these gains are wiped out in the last years of their working lives. This unusual result is caused by insufficient sample size. In the final years-since-graduation cell there are only three observations, with an extremely low mean income, and each of these observations receives a disproportionately large weight in the rate of return computations. If these three observations had averaged $10,000 instead of $4,080, the rate of return computed from these cell means would have been positive instead of −1.0.

TABLE 14

Mean Incomes for Cells Defined by Years since Graduation and Years of Graduate Education

Years since Graduation (i − 4)	Class Interval (years)	Years of Graduate Education					
		0		1 1/2		3 1/4	
		Mean	Number	Mean	Number	Mean	Number
0	1	3440	48	0	0	0	0
1	1	3531	39	2597	4	0	0
2	1	3180	25	0	0	0	0
3	1	3591	27	4397	4	3520	1
4	1	3634	103	3534	9	3840	2
5	1	3909	126	3416	12	4413	3
8	5	5115	485	4471	89	5898	53
13	5	6608	296	5338	87	7456	50
18	5	8238	229	6337	93	8294	36
23	5	10158	170	6889	46	9762	35
28	5	10206	96	7527	25	10859	16
33	5	10443	75	11093	18	8660	8
38	5	8507	46	5887	7	12459	14
43	5	10736	20	7646	5	4080	3

This shortcoming suggests that rates computed from an uncontrolled regression would be more accurate in cases of small sample size, since each computed cell mean would be derived from the whole sample rather than from just those few observations which happened to be in the particular cell. In the present case this advantage would appear to outweigh the danger

of misspecified regression form, and so the uncontrolled regression estimates of 2.2% to 3.0% will be taken as the better measures of the uncontrolled rate of return on investment in 3 1/4 years of graduate education.[1]

Both methods are in agreement, however, on estimating the return to investment in 1 1/2 years of graduate education at −1.0. This result comes from the fact that proportionately more people in education, the lowest paying of the occupations considered, are to be found among the holders of master's degrees.

Rates of return on graduate education, when controlled for other income determinants, are given in Table 15.[2] To speak in general terms, we may say that the return to investment on 3 1/4 years is 1 or 2 percent, but the return on 1 1/2 years is 0 to minus 1 percent. Therefore, when other income determinants are taken into account, the return to 3 1/4 years is changed very little, while the return to 1 1/2 years is raised drastically even though it remains very low. This demonstrates that a rate of return which is computed without taking into account these other determinants is not necessarily overstated. The direction of bias depends on the particular rate of return involved.

Implications of the Computed Rates of Return

In order to better evaluate the reliability of various rates of return which have been computed from regression equations, alternative computations were made using different values of the relevant education coefficients. The new values chosen were at the extremes of the 95% confidence interval of the education coefficient, i.e., the probability is .95 that two coefficient estimates thus chosen would bracket the true value of the education coefficient. A regression coefficient is a normally distributed variable, and so the two extreme values which define the confidence interval are symmetrical about the maximum likelihood estimate derived from the regression computations. As Table 16 shows, however, the rates of return computed from these extreme values are not similarly symmetrical. A high extreme value for the education coefficient cannot raise the rate of return greatly, but if a low extreme value is negative, i.e., if the maximum likelihood estimate deviates from 0 by less than 1.96 times its standard error, this suggests that more education results in less income, and the rate of return plummets to minus 100%.

---

1. To reduce the difficulties of too few observations in early-in-life cells where the class interval is only one year, all observations in these cells were pooled for both levels of graduate education and a single mean derived which was applied to each one-year cell.

2. These rates of return are based on income differentials among different education levels within a given occupation. As was described above with regard to Table 12, they are not suitable measures for comparing rate of return differences among occupations.

TABLE 15

Rates of Return—Postgraduate Education

(Controlled for Other Determinants)

Graduate/Undergraduate Expenditure Ratio— 2.0

Ability Level	150	125	100	75	50
	Returns on 3 1/4 years graduate education				
Business	.032	.024	.016	.008	.000
Government	.018				−.014
Education	.010				−.022
	Returns on 1 1/2 years graduate education				
Business	.006	−.002	−.010	−.016	−.024
Government	−.008				−.038
Education	−.014				−.044

Graduate/Undergraduate Expenditure Ratio— 1.5
Add .004 to above figures

Graduate/Undergraduate Expenditure Ratio— 3.0
Subtract .002 from above figures.

TABLE 16

Rates of Return—Confidence Intervals

	Maximum likelihood estimate	95% confidence interval of education coefficient
Expenditure per Pupil		
Uncontrolled regression	.120	.072 to .158
Controlled regression*	.050	−1.0 to .104
3 1/4 years Graduate Education**		
Uncontrolled regression	.024	−1.0 to .094
Controlled regression*	.016	−1.0 to .092
1 1/2 years Graduate Education**		
Uncontrolled regression	−1.0	−1.0 to −1.0
Controlled regression*	−.010	−1.0 to .064

*Computed for business at an ability level of 100.
**Graduate/undergraduate expenditure ratio is 2.0.

The wide range of rates of return which are possible within a 95% confidence interval demonstrates the need for caution in interpreting the results. With this in mind, we may derive the following implications.

Concerning the rate of return on improved quality of undergraduate education, the maximum likelihood estimate from the controlled regression is outside the confidence interval of the uncontrolled regression, and vice versa. The controlled rate of return, which attempts to isolate education effects from other effects on income, is therefore significantly lower than the uncontrolled rate. To judge from the maximum likelihood estimates which follow, introduction of controls has cut the rate of return by more than 50% and puts it below Becker's 8% estimate of the social rate of return on physical capital.[1] It should be remembered, however, that this 5% figure is subject to all the downward biases described earlier. It is a minimum figure, and therefore cannot be compared directly to Becker's 8% figure, with the quick inference that there has been overinvestment in human capital relative to physical capital.

The measures of the rate of return on investment in graduate education can be treated only as orders of magnitude, since the confidence intervals are so wide, but it seems reasonably clear that the returns are very low. As noted previously, this low return is a minimum figure, and does not necessarily mean that society has (or rather had, in 1947) overinvested in graduate education. It does suggest that if overinvestment has not occurred the social returns must come largely from external effects and nonmoney income, particularly in the form of more enjoyable work, since the money returns are so low. The results of this computation should serve as warning of the pitfalls which lie in the way of those tempted to argue for more education purely on the basis of money returns.

Finally, the results demonstrate that if rate of return computations do not incorporate controls for many if not all income determinants, the biases introduced can be phenomenally large. This is particularly obvious in the measurement of returns to 1 1/2 years of graduate education, where a return of minus 100% must be classed as a nonsense result.

8. Concluding Comments

Applying data to a purpose other than that for which it was collected is hardly ever a satisfying experience. The Time data provide no exception to this statement. The contrast between enumeration of income determinants and enumeration of measures actually available in the Time data should make its shortcomings obvious. Nevertheless it was the best source available. Other bodies of data suffered from even greater shortcomings—age structured in only three or four groups, or inadequate number of socio-economic variables which could be interpreted as income determinants, or no information on ability, a variable which seemed to be of particular importance.

---

1. Becker, op cit., p. 349.

The imperfections of the measurements mean that all statistical results must be accepted with caution. The regression forms chosen for use may be misspecified in an infinite number of ways. It is quite possible, for example, that race, social class, and many other characteristics are truly determinants of income even though the statistical analysis did not recognize their importance. The danger of having omitted significant variables from the regression form also calls into question the interpretation placed on relationships which were found to be significant. It may be, for example, that since ability is correlated with social class the positive ability coefficient involves an element of market imperfection which favors high social classes as well as a measure of actual labor productivity.

These comments serve to emphasize again that the measures provided by this study should be interpreted as orders of magnitude. They are nevertheless another step toward the goal of deriving measures directly applicable to questions of social policy. Until more accurate estimates are made by others, the present results may serve as a base from which estimates may be made of the quantitative impact of biases arising from the four assumptions set forth in the first part of this study.

In addition, the study has two other purposes. It is intended as an empirical inquiry into the determinants of earned income, independent of the implications such determinants have for measuring rates of return, and it serves to illustrate a method for deriving rate of return measures through regression equations.

Summary: This essay reports some experiments in oligopoly. A behavioral model is developed in which the decisions of a subject (firm) are related to the decisions made by the other subjects in his game (industry) and to certain characteristics of industry structure. It is found that the model explains reasonably well the subjects' behavior. In the first of the two experiments reported here, all games were duopolies, while in the second, there were two, three, and four person games. These experiments build on earlier work, primarily that of Lawrence Fouraker and the late Sidney Siegel. This work differs from theirs in its approach to the individual subject. The approach here is to model the behavior of each subject, rather than to predict an equilibrium game outcome independent of who is in the game.

This essay was submitted to Yale University in 1963 in partial fulfillment of the requirements for the degree of Doctor of Philosophy. The author is currently Assistant Professor of Economics at Yale University.

Dissertation Committee: Professors William Fellner (Chairman), John W. Hooper, and John F. Muth.

*

I would like to thank Dr. Martin Shubik for his encouragement and for initially arousing my interest in this area. I owe Professors Fellner and Hooper much for their help and advice; and Professor George Feeney, with whom I had many discussions, has affected this work in several ways. Several friends were especially generous: Laurence E. Lynn for his patience and help in many discussions and Mrs. Stephen Joseph for her help in carrying out the experiments.

J. W. F.

INDIVIDUAL BEHAVIOR IN OLIGOPOLISTIC MARKETS: AN EXPERIMENTAL STUDY

James W. Friedman

I. Introduction

The main body of this essay is concerned with oligopoly experiments carried on by the author in which Yale University undergraduates acted as subjects, each subject taking the part of an idealized business firm. The experiments were designed to provide data for the testing of particular hypotheses about the behavior of the subjects. The hypotheses tested were chosen largely as a result of previous experimental and theoretical work carried on by others.

Before describing the experiments, it will be helpful briefly to review some of the theory of oligopoly and some recent experimental work in order to place them in proper perspective. The essential difference between oligopoly, on the one hand, and monopoly and pure competition, on the other, is that the decisions of an oligopolist materially affect the profits, and hence the decisions, of the other firms in his industry. Similarly, the other firms' decisions affect the profits and decisions of the first firm. Theories of oligopoly attempt to take this interdependence into account.

A number of solution concepts have been put forth as explanations of how oligopolists resolve their decision problem or as prescriptions showing how they should resolve it. As a description of how oligopolists behave, no single concept has yet been shown to cover all circumstances; however, certain solutions have been found in experiments to describe behavior well in a limited number of cases. The experiments described below were designed to build upon experiments conducted earlier by L. E. Fouraker and the late Sidney Siegel.[1]

Three solutions which will be briefly reviewed here are the joint maximum, the noncooperative, and the cutthroat.[2] A "Chamberlinian" duopoly model will be used to illustrate the solutions. The sales of firm 1 are given by:

$$q_1 = a_1 - a_2 p_1 + a_3 p_2$$

and the sales of firm 2 are given analogously. Total cost for firm 1 is:

$$TC_1 = c_{11} + c_{12} q_1 + c_{13} q_1^2$$

---

1. Bargaining Behavior II (University Park: Pennsylvania State University, 1961).

2. For a discussion of additional solution concepts, including some based upon the Nash bargaining model, see J. P. Mayberry, J. F. Nash, and M. Shubik, "A Comparison of Treatments of a Duopoly Situation," Econometrica, XXI (1953), 141–54.

and for firm 2 is:

$$TC_2 = c_{21} + c_{22}q_2 + c_{23}q_2^2$$

The model is "Chamberlinian" because the form of the demand curve implies that products are differentiated and because prices are written as independent variables. Figure 1, below, illustrates this demand. The dd' curve results when p_2 is constant (at say p_2^*) while p_1 varies. dd' has a slope of $-a_2$. The DD' curve is had when $p_2 = p_1$ and the two prices vary together.[1] The DD' curve has a slope of $-(a_2 - a_3)$.

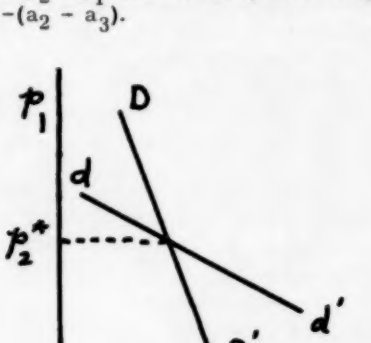


Figure 1

The profit levels of the firms are:

$$\pi_1 = p_1q_1 - TC_1 \qquad \pi_2 = p_2q_2 - TC_2$$

Joint, or total industry, profit is maximized when $\pi_1 + \pi_2$ is maximized simultaneously with respect to p_1 and p_2.[2] That is, when:

---

1. See E. H. Chamberlin, The Theory of Monopolistic Competition (7th ed.; Cambridge, Mass.: Harvard University Press), Chapter V.

2. $$\frac{\partial(\pi_1 + \pi_2)}{\partial p_1} = q_1 + p_1\frac{\partial q_1}{\partial p_1} + p_2\frac{\partial q_2}{\partial p_1} - \frac{\partial TC_1}{\partial q_1}\frac{\partial q_1}{\partial p_1} - \frac{\partial TC_2}{\partial q_2}\frac{\partial q_2}{\partial p_1} = 0$$

$$= a_1 + a_2(C_{12} + 2a_1C_{13}) - a_3(C_{22} + 2a_1C_{23}) - 2(a_2 + a_2^2C_{13} + a_3^2C_{23})p_1$$

[continued]

$$\frac{\partial(\pi_1 + \pi_2)}{\partial p_1} = 0 \qquad \frac{\partial(\pi_1 + \pi_2)}{\partial p_2} = 0$$

The noncooperative solution is a slight generalization of the well-known Cournot solution. Under the Cournot solution, each firm maximizes its own profit with respect to its own output level, while under the noncooperative, the firm maximizes with respect to whatever variable or variables are under its control. With the model used above, the firms each maximize profits with respect to their own prices:[1]

$$\frac{\partial \pi_1}{\partial p_1} = 0 \qquad \frac{\partial \pi_2}{\partial p_2} = 0$$

$$+ \; 2a_3(1 + a_2C_{13} + a_2C_{23})p_2 \frac{\partial(\pi_1 + \pi_2)}{\partial p_2} = a_1 + a_2(C_{22}$$

$$+ 2a_1C_{23})$$

$$- \; a_3(C_{12} + 2a_1C_{13}) - 2(a_2 + a_2^2C_{23} + a_3^2C_{13})p_2$$

$$+ 2a_3(1 + a_2C_{23} + a_2C_{13})p_1 = 0$$

Second order conditions require that: $\frac{\partial^2(\pi_1 + \pi_2)}{\partial p_1^2} < 0;$

$\frac{\partial^2(\pi_1 + \pi_2)}{\partial p_2^2} < 0$ and $\frac{\partial^2(\pi_1 + \pi_2)}{\partial p_1^2} \cdot \frac{\partial^2(\pi_1 + \pi_2)}{\partial p_2^2} - \left[\frac{\partial(\pi_1 + \pi_2)}{\partial p_1 \partial p_2}\right]^2 > 0$

---

1. $\frac{\partial \pi_1}{\partial p_1} = q_1 + p_1 \frac{\partial q_1}{\partial p_1} - \frac{\partial TC_1}{\partial q_1} \frac{\partial q_1}{\partial p_1} = 0 = a_1 + a_2(C_{12} + 2a_1C_{13})$

$$- \; 2a_2(1 + a_2C_{13})p_1$$

$$+ \; a_3(1 + 2a_2C_{13})p_2 \frac{\partial \pi_2}{\partial p_2} = a_1 +$$

$$a_2(C_{22} + 2a_1C_{23}) - 2a_2(1 + a_2C_{23})p_2$$

$$+ \; a_3(1 + 2a_2C_{23})p_1 = 0$$

Second order conditions require: $\frac{\partial \pi_1}{\partial p_1} < 0; \; \frac{\partial \pi_2}{\partial p_2} < 0.$

The cutthroat solution is found by solving:[2]

$$\frac{\partial(\pi_1 - \pi_2)}{\partial p_1} = 0 \qquad \frac{\partial(\pi_2 - \pi_1)}{\partial p_2} = 0$$

Here, the firm behaves so as to maximize the difference between his profit and the profit of the other firm.

There is a more general way of viewing all these solutions. This approach, shown to me by Dr. George Feeney, consists in viewing the objective of firm 1 as the maximizing of $\pi_1 + \rho_1\pi_2$ with respect to p_1 under the assumption that p_2 remains constant. $\rho_1 = \rho_2 = 1$ corresponds to the joint maximum; $\rho_1 = \rho_2 = 0$ corresponds to the noncooperative solution; and $\rho_1 = \rho_2 = -1$ corresponds to the cutthroat competitive solution. With the Feeney approach the firm need not be viewed as behaving in one of three ways. In fact if it is assumed that the firms behave as described above, then one can work backward from decisions to get the implied values of ρ_1 and ρ_2. The analysis of the experiments makes use of Feeney's approach.

In addition to the problem of modelling behavior, there is the problem of modelling cost, demand, and interdependence of firms in an industry. To a large extent, these problems are the same whether the industry is competitive or monopolistic. There are, of course, some special problems involved in modelling oligopoly which are related to the interdependence of firms. The history of modelling oligopolistic industries dates from A. A. Cournot. This history will be reviewed very briefly, discussing only those developments most germane to the experiments.

---

2. $$\frac{\partial(\pi_1 - \pi_2)}{\partial p_1} = a_1 + a_2(C_{12} + 2a_1C_{13}) + a_3(C_{22} + 2a_1C_{23}) - 2(a_2 + a_2^2C_{13} - a_3^2C_{23})p_1 + 2a_3(a_2C_{13} - a_2C_{23})p_2 = 0$$

$$\frac{\partial(\pi_2 - \pi_1)}{\partial p_2} = a_1 + a_2(C_{22} + 2a_1C_{23}) + a_3(C_{12} + 2a_1C_{13}) - 2(a_2 + a_2^2C_{23} - a_3^2C_{13})p_2 + 2a_3(a_2C_{23} - a_2C_{13})p_1 = 0$$

Second order conditions require: $\frac{\partial^2(\pi_1 - \pi_2)}{\partial p_1^2} < 0$ and $\frac{\partial^2(\pi_2 - \pi_1)}{\partial p_2^2} < 0$

Cournot was a mathematician whose well-deserved fame in the history of economics rests upon one book, Researches into the Mathematical Principles of the Theory of Wealth.[1] Chapter VII deals with oligopoly. Most detailed attention is given to duopoly and attention here will focus on duopoly. Cournot postulates two entrepreneurs, each of whom owns a mineral spring. The springs are identical and, if their outputs are denoted respectively by q_1 and q_2, demand for their product is given by $p = f(q_1 + q_2)$. $\frac{\partial p}{\partial q_1} = \frac{\partial p}{\partial q_2} < 0$. Costs are nil for both producers; hence it is the desire of each to maximize his total revenue:

$$R_1 = pq_1 = q_1 f(q_1 + q_2)$$

$$R_2 = pq_2 = q_1 f(q_1 + q_2)$$

Cournot comments that if the two producers collude to obtain jointly maximal profits, the results, so far as the consumers are concerned, will not differ from those of monopoly. As Cournot had treated monopoly in Chapter V, he explored another solution: noncooperative equilibrium. This solution is a special case of the noncooperative equilibrium discussed above in that it is quantity which is the decision variable. Previously, the decision variable was not specified.

It is well known that Cournot's work passed unnoticed until 1883 when another French mathematician, Joseph Bertrand, reviewed Cournot's book.[2] Only one paragraph of the review is concerned with the seventh chapter of Cournot. He objects to Cournot's casting the joint maximum solution aside; moreover, he claims that Cournot's analysis is entirely wrong even if the firms behave noncooperatively.

In Chapter V ("On Monopoly") Cournot had treated price as the decision variable while in Chapter VII he so treats quantity. His results are correct for his model with quantity treated as the decision variable; however, entirely different results obtain if price is treated as the decision variable. The two producers are dealing with a perfectly homogeneous good and rational customers; therefore, if one firm charges less than the other, the firm charging the lower price can sell all the customers in the market. In the mineral spring

---

1. Antoine-Augustin Cournot, Researches into the Mathematical Principles of the Theory of Wealth (New York: Kelley and Co., 1960). The "Researches" originally appeared in 1838; English translation, prepared by N. T. Bacon, appeared in 1897.

2. Joseph Bertrand, "Review of Walras' Théorie Mathématique de la Richesse Sociale and Cournot's Recherches Sur les Principes Mathématiques de la Théorie des Richesses," Journal des Savants, Vol. LXVIII (1883), 449–508.

case this implies a noncooperative equilibrium price of zero. Edgeworth generalized the Bertrand case to variable costs. We shall not go into this.

The next major step was the modification proposed by Hotelling and Chamberlin.[1] Briefly put, if the demand faced by firm i in an industry composed of n firms is given by:

$$q_i = f_i(p_1, \ldots, p_n)$$

where f_i is a twice differentiable function and $\frac{\partial q_i}{\partial p_j} > 0$, $\frac{\partial q_i}{\partial p_i} < 0$ and $\sum_{j=1}^{n} \frac{\partial q_i}{\partial p_j} < 0$. That is, the demand curve facing a firm, given all other firms in the industry keep their prices constant, is downward sloping; and the firm's demand curve, if all other prices change with his own price, is downward sloping and steeper.

Hotelling's formulation and some of Chamberlin's graphs employ a linear demand function, such as that in Figure 1 above. Algebraically this could be written as:

$$q_i = a - bp_i + c\bar{p} \qquad b > c \qquad a, b, c, > 0$$

The Hotelling-Chamberlin modelling of oligopoly with price as decision variable seems to me much more generally valid than the Bertrand-Edgeworth model. The latter may be had as a special case of the former if (b-c) remains constant as b and c go to infinity.

While Chamberlin, Triffin, and, more recently, Shubik have written on the question of entry into and exit from an industry, and work on the strategic aspects of oligopoly has been carried on as an application of the Nash bargaining model, these topics are not necessary to the understanding of the experiments described below or those described by others; therefore, we will proceed to a brief discussion of the Fouraker-Siegel experiments upon which the author's experiments build.

II. Experimentation in Economics

The use of experimentation in economics is comparatively new, the earliest examples of which I am aware being E. H. Chamberlin's experiment in bilateral monopoly, published in 1948, and Mosteller and Nogee's experi-

---

1. Edward Chamberlin, Theory of Monopolistic Competition (7th ed., Cambridge, Mass.: Harvard University Press, 1956), and Harold Hotelling, "Stability in Competition," Economic Journal, XXXXI, 41-57.

ment in utility, published in 1951.[1] It has often been said that experimentation is impractical for economics:[2]

> It is a commonplace that, in its choice of method, economics is limited by the fact that resort cannot be had to the laboratory techniques of the natural sciences. On the one hand, the data of real life are necessarily the product of many influences other than those which it is desired to isolate—a difficulty which the most refined statistical methods can overcome only in small part. On the other hand, the unwanted variables cannot be held constant or eliminated in an economic "laboratory" because the real world of human beings, firms, markets, and governments cannot be reproduced artificially and controlled. The social scientist who would like to study in isolation and under known conditions the effects of particular forces is, for the most part, obliged to conduct his "experiment" by the application of general reasoning to abstract "models." He cannot observe the actual operation of a real model under controlled conditions.
>
> The purpose of this article is to make a very tiny breach in this position . . .

Whatever the size of the breach made by Chamberlin in that article and by Mosteller and Nogee three years later, it has been growing to the point that, today, there is a real possibility that experimentation will prove a most valuable source of new knowledge and insight.

Economics has both a normative and a descriptive side. The descriptive side may be very abstract in the sense that it often concerns itself with the description of very idealized economic situations and processes—which sometimes cannot be found in "real" life.

For example, Chamberlin's contention that firms in an oligopolistic industry will jointly maximize profits when there is complete information and the firms are symmetric is presumably a descriptive proposition; however, it describes a highly simplified situation which occurs rarely, if ever, in life. Similarly, from the experiments described later in this chapter and in later chapters, generalizations have been made about the behavior of

---

1. Edward Chamberlin, "An Experimental Imperfect Market," Journal of Political Economy, Vol. 56 (1948), pp. 95–108, and F. Mosteller and P. Nogee, "An Experimental Measurement of Utility," Journal of Political Economy, Vol. 59 (1951), pp. 371–401.

2. Edward Chamberlin, op. cit., p. 95.

subjects which summarize and draw on their actual behavior. These analyses of experimental data purport to describe the behavior of certain classes of individuals in a highly simplified experimental context.

The reason for conducting experiments is that we have extremely little data on individual behavior which we are able to analyze with fruitful result. By designing and conducting experiments, the economist generates data himself which is relevant to problems in which he is interested. The advantages of experimenting, instead of using empirical data generated by government, industry, etc. in their everyday tasks, are several. Empirical data are not always available because records have not been kept or have not been kept in a form particularly useful to the economist. Thus, for example, the value at which a machine is carried on the books of a firm rarely reflects either its present productive power relative to a new model or its current market value; however, these are particular values in which an economist would be interested. Often empirical data are incomplete and analysis based on them must be either incomplete or based upon complete data where the holes have been filled by ad hoc methods.

Experiments conducted in oligopoly have been concerned with the behavior of firms and the extent to which the decisions of firms lead to noncooperative equilibrium or other of the known solution points. Here empirical data would be extremely difficult, if not impossible, to use. Given whatever information the four major automobile companies, say, have on their individual costs, demand, records of decision on prices, advertising, selling effort, etc., how does one go about finding any of the equilibrium points? Just considering the demand for automobiles, the effects of trends in disposable income, cyclical fluctuations, age distribution of the existing stock, and variety of sizes and models must be taken into account if only a ten-year period is studied.

Experimentation, on the other hand, has the advantage that the experimenter can design, to suit his purposes, many of the features of the environment in which the subject is to operate. This is both the strength and the weakness of experimentation. As a strength, it means the experimenter in oligopoly can use a model in which equilibrium points in which he is interested are known and unique. If he wants to study the effect of certain changes in the environment, while holding it constant in other respects, he is free to do so. For example, one might wish to see if the tendency to noncooperative equilibrium is stronger when firms (subjects) cannot communicate with one another than when they are allowed to send written notes back and forth.

With oligopoly experiments, there are many elements which come to mind whose effects are probably worth studying. First, variations in the amount and type of contact between subjects could influence behavior. If subjects are allowed no personal contact or communication, there remains the completeness with which results of past periods of play are reported to them. They may, for example, be told their own profits at the end of a period of play and be given anything from no information on the decisions or outcomes of

competitors to full information. Their information at the beginning of play might consist of complete specification of the model and parameter values which underly the experimental situation, or the initial information might range down to the knowledge that one must make a price decision and one has certain costs—without a word on demand or the cost structure of other firms.

Similarly, there might be a wide range of personal contact between those in the same "industry." At one extreme, none is allowed. If it is allowed, probably the best means is written notes. Using notes, the experimenter has a record of all communication and, if he wishes to limit the range of discourse and type of information which is exchanged, he can play delivery boy and censor the notes to be sure they conform to the standards which he set out for the subjects. Also, side payments could be explicitly allowed or disallowed.

Behavior might be found to vary with certain characteristics of the structure of the model. The level of profit involved in the game and the relative increase in profits accruing to the firm if the industry switched from noncooperative equilibrium to, say, joint maximization might be relevant variables. The number of firms in the industry and the complexity of the model as reflected in the number of decision variables might also be important characteristics of structure.

As subjects gain experience in experimental games, their modes of play and the consequent outcomes of their decisions could change. Probably, it is the behavior of experienced subjects which is of greatest relevance in oligopoly; for, certainly those in business who make decisions are experienced and have the "feel" of their particular jobs which comes with experience. As for subjects, early in their experience, they are more likely to be influenced by the tone of the instructions and other extraneous matter than they are after they have some history and experience in the experiment itself. After experience has been gained the subject will probably tend to infer (rightly or wrongly) from it about how he ought to behave.

The subject's reactions to the decisions of other subjects would seem a most promising area to explore. Would subjects who played noncooperatively when their competitors did so continue to play noncooperatively if their competitors began playing cooperatively? In general, would the cooperativeness of a subject depend upon the cooperativeness of the others in his industry (game) or would it be independent?

Finally, it would certainly be interesting to see if behavior differed when the subject was, on the one hand, in the position of an owner-manager (the classical entrepreneur) or, on the other hand, in the position of a non owning manager. On the significance of the professional manager in our economy, Berle and Means, in their famous book, write:[1]

1. A. A. Berle and G. C. Means, The Modern Corporation and Private Property (New York: Macmillan Co., 1933).

> Corporations have ceased to be merely legal devices through which the private business transactions of individuals may be carried on. Though still much used for this purpose, the corporate form has acquired a larger significance. The corporation has, in fact, become both a method of property tenure and a means of organizing economic life. Grown to tremendous proportions, there may be said to have evolved a "corporate system"—as there was once a feudal system—which has attracted to itself a combination of attributes and powers, and has attained a degree of prominence entitling it to be dealt with as a major social institution.[1]
>
> . . . . . .
>
> Though the American law makes no distinction between the private corporation and the quasi-public, the economics of the two are essentially different. The separation of ownership from control produces a condition where the interests of owner and of ultimate manager may, and often do, diverge, and where many of the checks which formerly operated to limit the use of power disappear.[2]
>
> . . . . . .
>
> ...the shareholder has a piece of paper with an open market value, and as holder of this paper may receive from time to time, at the pleasure of the management, periodic distributions. He is forced to measure his participation, not in assets, but in a market quotation; and this market quotation "discounts" or appraises the expectation of distributions.[3]

These comments raise many interesting and important questions; however, only one will be pursued here. Does the professional manager behave differently than the owner-manager? Some attempt to get at this experimentally might be made by designing an experiment in which there were two (or any even number) of separate industries, both with the same number of firms. Numbering the industries 1, 1', 2, 2', etc. and the firms within any industry A, B, ..., the subject who makes decisions for firm 1A would be told how his payoff relates to the profits of his firm. For example, he might get a

---

1. Ibid., p. 1.
2. Ibid., p. 6.
3. Ibid., p. 287.

salary (i.e., a fixed amount) for each period of play and a bonus of 10 per cent of firm profits above some given level π_0. He might further have to undergo salary cuts or lose salary altogether if the firm loses money for several successive periods. Finally, so that he knows there are really "stockholders" who get the profits of the enterprise, he is told that one of the managers of a firm in an industry other than his own gets the profits of the firm, and he gets the profits of some certain other firm in a different industry. The manager of 1A might be the "owner" of 1'A, and vice versa. The subject would never meet his "employer" or his "employee"; however, as a manager, he would be aware, as real managers are, that stockholders exist to whom he presumably has a responsibility, but who cannot effectively remove him from office.

In a sense, the very strengths and advantages of the experimental method are also the source of its limitations. The experiment is purposely limited in scope and simple in structure so that the experimenter may be capable of analyzing his model, isolating certain characteristics in a particular experiment, and analyzing the behavior of his subjects in terms of the chosen characteristics. Daily life is complex, containing many of the features and complications explicitly left out of the experiment. Do these omissions vitally affect experimental results? With regard to oligopoly experiments, some of the important considerations are: Is the motivation of subjects in the game in some sense equivalent to the motivation of businessmen in business. Economic theory typically holds that businessmen seek to maximize their profits, and that, if profit maximization is not the sole goal, it is the dominant one. In experimental games, in order to create a meaningful opportunity for profit maximization, the subject is told his remuneration for participation in the game will be the actual profit he earns while playing. The amounts of money involved in the game are chosen to be large enough that the difference in profit corresponding to various outcomes (say, joint maximization and noncooperative equilibrium for example) will be significant and important to the subjects. Now, to the extent that other forms of motivation which do not imply profit maximizing behavior are important, the subjects' motivation in experimental games misses the mark of being a good representation of the motivation of businessmen in business. H. A. Simon has suggested that entrepreneurs are satisficers, not maximizers.[1] Instead of trying to make profits as large as possible, they try to make them stay at or above a particular level with which they are content. Baumol suggests an entrepreneurial goal somewhat like that of Simon: "...once [the businessman's] profits exceed some vaguely defined minimum level, he is prepared to sacrifice further increases in profits if he can thereby obtain larger revenues."[2]

---

1. H. A. Simon, "Theories of Decision Making in Economics and Behavioral Science," American Economic Review, Vol. 49 (1959), pp. 253-83.

2. W. J. Baumol, "On the Theory of Oligopoly," Economica, Vol. 25 (1958), pp. 187-98.

The difference in time horizon between the businessman and the experimental subject may have an important bearing on behavior. Businessmen do not usually know they have only a short time left before they will be out of business. Even if a man is about to retire, he knows the business will go on after he leaves, and he will probably conduct himself as if he were to be in it for an indefinite time period. On the other hand, the subject knows approximately how long he will be participating in an experiment and he can guess the number of game periods which he will play. He may act as if he were planning on getting out of "business" in a short time. The knowledge that the end is near might cause less cooperative behavior than the assumption that the end is in the indefinite future.

There may also be important differences between the typical subjects of experiments and businessmen. Subjects tend to be undergraduate students who are looking for part-time work. If the behavior of businessmen is greatly affected by certain characteristics which they typically have and which undergraduates generally do not, then comparisons drawn between undergraduate subjects and businessmen will be the more inaccurate.

We cannot now draw inferences from experimental games about real markets, not because the two are not comparable, but because we do not know whether or not the two are comparable. At the least, continued research should yield information on how people behave in certain types of economic situations (the games themselves, and any economic situation which closely resemble them). It is also possible that as our experience and ability increase, we will be able to model certain types of markets closely enough that inferences may be drawn from games to markets.

Some interesting experimental work has been done in recent years. We will look at a group of experiments by L. E. Fouraker and S. Siegel.[1]

They conducted ten oligopoly sessions. Six utilized Cournot type quantity games, four used Bertrand type price games. Two of the quantity game sessions have special characteristics which make them considerably different from the author's experiments. In each of the ten sessions, subjects participated in exactly one game and no subject participated in more than one session. The sessions were conducted so that subjects would never know the identity of the other players in their game, and no communication was allowed between players. Before making a decision for a period, subjects were given the results of the preceding period.

The first four sessions are the only ones to be reviewed here. They were, respectively, incomplete information duopoly, incomplete information triopoly, complete information triopoly, and complete information duopoly. Under complete information, the subject was given a payoff matrix, the columns of which were labelled with the quantity choices available to the subject, and the rows were labelled with the sums of possible quantity choices

1. L. E. Fouraker and S. Siegel, Bargaining Behavior II (University Park: Pennsylvania State University, 1961).

available to the other players. The body of the table contained the profit to the subject and the sum of the profits to the other players corresponding to any combination of quantity choices they might make. In the case of triopoly, the subject was told the individual quantity choices of the other two players and their individual profits. Under incomplete information, the payoff matrix was labelled just as under complete information; however, the body of the table contained only the profit to the subject corresponding to any combination of production choices. With triopoly, the subject was given only the total production of his two competitors and was told nothing about their individual or joint profit.

Each game consisted of 25 periods, the first three of which were called A, B, and C and were "practice periods," which meant the subjects had no monetary gain or loss on them. The remaining 22 periods, numbered sequentially from 1 to 22, were "regular" periods and the subjects were paid the profit they earned in them.

The model used for these games was extremely simple—literally a Cournot "mineral spring" model. The firms had no costs and faced a demand curve given by:

$$p = A - B(q_1 + q_2 + \ldots)$$

A was set equal to 2.4 (cents) and B equal to 0.4. The players could choose any integral quantity from 8 to 32.

Both sets of incomplete information experiments show strong convergence to the Cournot equilibrium, which is a production rate of 20 units per firm in duopoly and 15 per firm for triopoly. Fouraker and Siegel expected this and explained it by pointing out that even if a player wanted to be cutthroat or cooperative, he could not be sure how to do so, because he lacks knowledge of the cost structures of the other players.

Their method of analysis for the incomplete information games was to calculate the aggregate production level in each game for a typical period for each of the sessions, compute the mean aggregate production for each of the two samples, and test to see if these means differ significantly from the Cournot and from the Pareto optimal quantities. The 21st period was used as data. They found the means did not differ significantly from the Cournot, but did from the Pareto optimal. There were 16 incomplete information duopoly games, the same number of complete information duopoly games, and there were 11 triopoly games of each type.

The data from the two complete information sessions was analyzed in a somewhat different manner. The two triopoly sessions were compared with one another, and the two duopoly sessions with one another, to test whether or not the production decisions showed more variability under complete information than under incomplete information (from game to game) using the 21st period as data. It was found that it did in both cases.

Their use of the next to last period, alone, as data seems to me a poor

idea. A glance at the players' decisions, especially in the complete information games, will show that there is considerable randomness (or at least unexplained variation) in them; hence, any single period might be quite typical of the game as a whole or of the later periods. This would not be serious if the sample were large, say a couple of hundred games, but where the number of games is only 11 or 16, the result could be that the data taken from one period per game result in distortion of the distribution of overall outcomes between cooperative, noncooperative, and competitive (the three classifications used by Fouraker and Siegel). Presumably Fouraker and Siegel were interested in the behavior of the subjects after they had gotten used to the game and to the other players with whom they were paired. It might have been better for them to use the five or ten periods preceding the end as their data.

III. Background of the Two Series of Games and Elements Common to Both

Looking at the four oligopoly sessions held by Siegel and Fouraker, three conclusions may be drawn from their data: When players were under the "incomplete information" condition, their behavior was quite consistently noncooperative. Under complete information, the triopolies were also quite consistently noncooperative; but the duopolies showed a great variety in outcome. Some of them were cooperative, some were noncooperative, some were competitive, some were in between, and some showed no tendency to stabilize. Fouraker and Siegel made the suggestion that differences in individual characteristics of the subjects caused the great divergence in game results under complete information duopoly.

They put forth three (polar) types of individual: the "simple maximizer" who tries to maximize his profit noncooperatively; the joint maximizer, who, as his desription implies, tries to maximize industry profits; and the rivalistic player, who tries to maximize the difference between his profits and those of the other player(s). The title "simple maximizer" suggests naiveté and makes one wonder if the joint maximizer is, in Fouraker and Siegel's minds, a sophisticated maximizer. If so, the sophisticated maximizer, interested really in his own profit, might be willing to jointly maximize only if the other players will similarly cooperate.

Their suggestion of individual differences seemed plausible and important enough to warrant testing, so the experiments reported in the remainder of this dissertation were expressly designed to examine individual behavior. Instead of describing a player as one of the three categories named in the preceding paragraph, it is more general, and probably more useful to describe subjects in terms of Feeney's rho. Thus, a subject need not be presumed to be at one of the two extremes (rho of 1 or -1) or midway between them. He can be anywhere along a continuum which would generally be bounded by 1 and -1.

The question of whether the joint maximizer is in fact interested only in his own profit and taking a "long range" view or is in fact really somewhat

altruistic relates to the kind of rho he will play. If he is really interested in himself, he may not play a high rho if the others in the game do not; however, if his real intent is to maximize industry profits, he will probably play a rho of 1, no matter what the other players do. The intent of a player is usually far from obvious from the way he plays. There may be several quite different intents which, given a game of some particular structure, will imply exactly the same behavior. It is not my purpose to presume we can generally tell the true intent of a player; however, it does not seem unlikely that some players will play different values of rho, depending on the way the others in the game play. Even though we cannot get at intent, there may be regularities in the way individual subjects play which are related to the way the others in the game play and to characteristics of game structure and the information conditions of the game. If the subject's play can be characterized by rho, we may say he plays as if he wishes to maximize his own profit plus rho multiplied by the sum of the profit of the other firms.

Two characteristics of game structure which I would imagine to be important—if any are—are "payoff level" and "relative payoff." The payoff level of a game is a value which gives an indication of the kind of money at stake. To approximate this, I have chosen the payoff level for player i, L_i to be the payoff to player i when the game is at noncooperative equilibrium. The relative payoff with respect to player i, R_i, is intended to be an indication of the relative differences in payoff from one equilibrium point to another. The two equilibrium points chosen are the joint maximum and the noncooperative equilibrium. Thus, R_i equals the ratio of player i's payoff when the game is at the joint maximum to noncooperative payoff, minus 1.

$$R_i = \frac{\pi_{iJM}}{L_i} - 1 \qquad (i = 1, \ldots, n)$$

where π_{iJM} is the profit firm i gets when the whole industry is at the joint maximum. Certainly, a game, even a simple one involving only prices and quantities, cannot be perfectly characterized by only two or several variables like L_i and R_i, but several well-chosen variables may do a rather good job of characterizing the structure of a game.

If the players in a game are in asymmetric positions with respect to profit, it may be necessary to introduce a third variable which gives an indication of the threat potential of the various firms.

The information structure of a game has two important dimensions. They are the amount and type of information which the player gets as feedback and the amount and type of communication the players are allowed to have with one another. The way both of these information conditions are set could affect the mode of play of the subjects. Fouraker and Siegel always kept the second of these types at the same level—no communication between players (by way of messages). The first type, the amount of information fed

back, had one of two states ("complete" and "incomplete" information) which were described previously. It was apparent from their experiments that the state was related to the outcome of the games, and to the way subjects played.

The model used in my experiments utilized a Chamberlinian demand function of the form:

$$q_i = a_1 - a_2 p_i + a_3 \bar{p} \qquad (i = 1, \ldots, n)$$

where $a_1, a_2, a_3 > 0$ and $a_2 > a_3$. The total cost function was of the form:

$$C_i = c_1 + c_2 q_i + c_3 q_i^2 \qquad (i = 1, \ldots, n)$$

where $c_1, c_2, c_3 > 0$. Marginal cost is, therefore, a linear increasing function of production.

A Chamberlinian price game was chosen in preference to a Bertrandesque price game because, in a Chamberlin price game, rho is a smoothly changing function of the prices of the players over a large interval which exceeds the interval from −1 to 1. By contrast, a Bertrandesque price game has discontinuities. The price choice for a player who wants to play a rho less than 0 is to charge a lower price than any other firm, or charge 0, whichever is greater. If we want to examine individual behavior in terms of rho, it is desirable to use a model in which values of rho are well defined and in which they smoothly vary as the values of the decision variables vary. In contrast to a Cournot type quantity model, the Chamberlinian price model may be adjusted so that the sensitivity of one firm's demand to decisions of the other firms relative to the sensitivity of its demand to its own decisions may be varied. This relative sensitivity is governed by the ratio of a_3 to a_2.

In modelling individual behavior, we have first to define the basic variable, rho. It will be defined analogously to the way it was defined in the preceding section: Given that

$$\rho_i = -\frac{\partial \pi_i / \partial P_i}{\sum_{j \neq i} \partial \pi_j / \partial P_i} = \phi(P_1, \ldots, P_n) \qquad (i = 1, \ldots, n)$$

ρ_{it} is defined by:

$$\rho_{it} = \phi(p_{1,\, t-1}, \ldots, p_{i-1,\, t-1}, p_{it}, p_{i+1,\, t-1}, \ldots, p_{n,\, t-1})$$

We assume the player acts as if, in choosing a price for period t, he is really choosing his desired value of ρ, and he chooses it on the assumption that the other players will charge the same price in period t as they did in period t−1.

$$\rho_{it} = \alpha_{i0} + \frac{\alpha_{i1}}{n-1} \sum_{j \neq i} \rho_{t-1} \qquad (i, j = 1, \ldots, n)$$

The meaning of this equation is that the player behaves as if his desired value of ρ_{it} equals some constant, α_{i0}, plus another constant, α_{i1}, times the average of the rho values of the other players for the preceding period. It seems reasonable to me that in games where there is no possibility of entry or exit, the parameters α_{i1} should be between 0 and 1. It does not seem reasonable that player i would lower ρ_i when player j raises ρ_j, nor does it seem reasonable that a player would raise his rho at a faster rate than the rate at which the average of the rho's of the others rises. Of course, only the gathering of experimental data will confirm or disprove these guesses about the probably limits of the α_{i0} and α_{i1}. In general, I would expect the limits of rho to be 1 and −1. Therefore, if α_1 lies between 0 and 1, I would expect α_0 to lie between $(1 - \alpha_1)$ and $-(1 - \alpha_1)$.

The other variables included in the behavioral equation are L_i, R_i, and n (the number of players). They are included in the following ways:

$$\ell_{it} = \beta_{i1} + \beta_{i2}L_{it} + \beta_{i3}R_{it} + \beta_{i4}D_{ilt} + \beta_{i5}D_{i2t} + (\beta_{i6} + \beta_{i7}L_{it} + \beta_{i8}R_{it} + \beta_{i9}D_{ilt} + \beta_{i10}D_{i2t}) \frac{1}{n-1} \sum_{j \neq 1} \ell_{j,\,t-1}$$

$(i, j = 1, \ldots, n)$. D_{ilt} = 1 when the number of players in the game is 3
= 0 otherwise

D_{i2t} = 1 when the number of players in the game is 4
= 0 otherwise

Hence:

$$\alpha_{io} = \beta_{i1} + \beta_{i2}L_i + \beta_{i2}R_i + \beta_{i4}D_{il} + \beta_{i5}D_{i2}$$

and

$$\alpha_{il} = \beta_{i6} + \beta_{i7}L_i + \beta_{i8}R_i + \beta_{i9}D_{il} + \beta_{i10}D_{i2}$$

In representing n with dummy variables in the expressions for α_{i0} and α_{il}, constant terms are independently estimated for the three values of n which occur in the games. The constant terms for n = 2, 3, and 4 in the expression for α_{i0} are, respectively, β_{il}, $\beta_{il} + \beta_{i4}$, and $\beta_{il} + \beta_{i5}$. In the expression for α_{il} they are β_{i6}, $\beta_{i6} + \beta_{i9}$, and $\beta_{i6} + \beta_{i,10}$. No variables representing information conditions are included because only one information state was employed—complete information in the sense Fouraker and Siegel use the term, with no messages or verbal communication allowed.

I conducted two series of games, each having its own group of subjects, its own payoff matrices and experimental design; however, certain procedures, which will be described below, were common to both series. In both series, the subjects were male undergraduate students of Yale University who were recruited through the Financial Aids Office of the University. The students were hired to perform "clerical work" at $1.50 per hour (which is about the going rate). Each student was under the impression that he was the only person hired—that only one was needed. Any student interested in the job after being told the pay and the hours of work was given my name and phone number. I repeated the pay and hours to the callers and answered any questions they had. Their questions were about the nature of the clerical work. I answered the work involved "getting some data for my thesis into order," which satisfied the curious. Any caller was hired if he wanted the job and was available during the required hours. The people at the Financial Aids Office were aware from the beginning of recruiting that the students would be in an experiment. They were very cooperative and tried to avoid hiring boys whose roommates had already been hired or boys who were known to have participated previously in economic experiments.

There are other methods which might have been used to recruit subjects. For example, notices might have been posted on bulletin boards which asked for people interested in "participating in a research project in economic decision making" or students in economics or other classes might have been asked if they wanted to participate in such a project. The most compelling reason for recruiting from the Financial Aids Office (which is where students go looking for term time employment) is that subjects gotten through it have the best chance of needing the money they expect to earn and they are more likely than the average student to have a strong profit maximizing motivation. Students recruited via bulletin boards and announcements in classes who are attracted to the "decision making project" may include the wealthy or the frivolous, for whom the money involved in the games is secondary or even totally unimportant. Frequently, if not invariably, players are strictly competitive when they are receiving either no pay for playing or an amount of pay which is unimportant to them. When there is no money, about the only objective left to the player is that of <u>doing better</u> than the others with whom he is playing.

The payoffs in each series were chosen so that if all played strictly noncooperatively, profits would come out to about $1.70 per hour. It was hoped that this general level of payoffs would be sufficient to motivate subjects drawn in the manner described. Advertising for clerical workers rather than for people to work at something which might sound interesting also had the purpose of attracting subjects for whom the money to be earned was of prime importance.

The games were held in a room which is part of the psychology department. Its layout is drawn in Figure 2, below. Around the sides and back wall of the room, forming a U, were nine cubicles, each about five feet wide and about eight feet long. The partitions between cubicles and between the cubicles

and the center of the room were about 6 1/2 feet high and curtains were hung on the doorway to each cubicle, so subjects could not see outside. Subjects were met individually at the entrance to the building in which the games were held and were escorted to the experimental room and into a cubicle. Whenever a group of games had ended, the subjects were allowed to leave singly, at intervals of several minutes. The purpose of these practices was to hold contact between subjects to a minimum. It was hoped that they would never see one another; however, we were not that fortunate. The subjects participating in the first series each played in six games, two on a Friday evening, two the following morning, and two on Saturday afternoon. Therefore, they entered and left the building three separate times, before and after each group of two games. The nine games of the second series were split into four groups, taking place on a Thursday evening, the following evening, Saturday morning, and Saturday afternoon. In both series, it was emphasized to the subjects that they should not discuss the games with anyone during the several days they took place, and preferably they should not discuss them later, in order to avoid biasing the results of these and future experiments.

The games varied in length from 22 to 26 periods. All periods were "regular" in that subjects were paid the profit they earned on them. Subjects were never told in advance how long games would last, and the game length was varied in the hope that they would always play under the assumption that there would be a number of periods to go—except for the final period of the game. Just before the results of the next to last period were handed back, it was announced that the next period (for which decisions were about to be made) would be the last. This announcement was made, of course, to bring out end effects. The subjects were paid their earnings on all games of the series after the last game was played.

After being escorted to a cubicle, each subject was given a set of instructions and asked to read them. He was told he could either participate in the project described in the instructions or do clerical work, and someone would come back to him in a few minutes for his decision. It turned out that no subject in either series refused to participate in the games. The player pairings were determined before each series; hence it was known, for example, with whom player A would be paired in each game of a series. However, the assignment of a company letter to each subject was done randomly. In the first series, a set of cards were made with one letter on each, the letters going alphabetically from A through H. After all subjects had arrived and decided whether or not to participate, someone went from cubicle to cubicle and had each subject draw one card from the pack. The card a subject drew gave his company letter. The process was the same in the second series, but the letters went from A through I.

The instruction sheet explained the manner in which the games would progress. It was written in such a way that, it was hoped, the subjects' behavior would not be biased toward any particular type (such as competitive, noncooperative, etc.). Words like "opponent" and "competitor" were avoided.

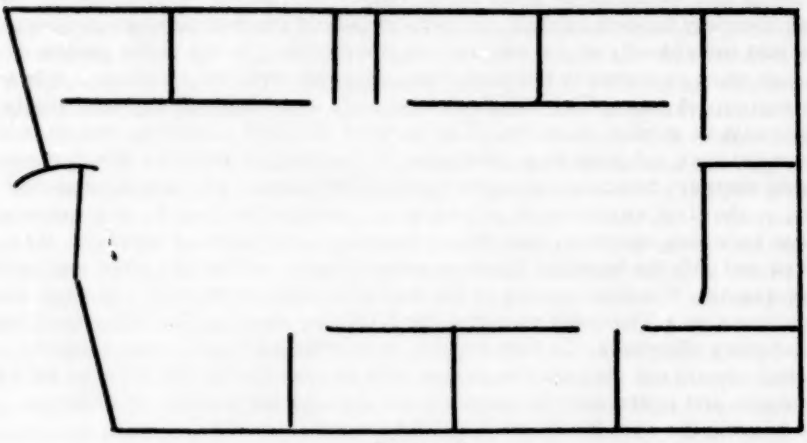


Figure 2

The other player(s) in a game were referred to as "the other seller(s)." The word "project" was used instead of "experiment" or "games" in the hope that its connotations were more serious to the subjects.

The subjects had a set of small sheets called "transmittal sheets" on which they were to record their price decisions for a period, the number of the period, the number of the game, and their company letter, each in its appropriate place. These were gathered by experimental assistants and split into groups corresponding to the player pairings for the current game. On each transmittal sheet was recorded the price charged by each of the other players, the average of the other players' prices if the game were 3 or 4 person, and the profit earned by the subject himself. The sheets were then returned and the subject recorded all the information on a master "record sheet" which he kept for the game. A short while later, the decision for the next period would be collected.[1]

The payoff matrices for the first series are "double entry." That is, for any price a player might choose (read down the side of the matrix) and any price the other player might choose (read along the top of the matrix), the body of the matrix contained two entries. One showed the profit to the

---

1. In the appendices to the dissertation upon which this essay is based are copies of the transmittal sheets, record sheets, instructions, payoff matrices, and complete game results. The materials for the first series are in Appendix A, and those for the second series are in Appendix B. See James W. Friedman, Individual Behavior in Oligopolistic Markets: An Experimental Study (New Haven: Yale University, 1963), Appendices A and B.

player, and the other showed the profit to his competitor. The payoff matrices for the second series were single entry. The columns were labelled with the average of the prices charged by the other players, and the matrix entries showed only the profit to the subject himself; however, it was emphasized that all players in the game had identical matrices. As a result, the player could always determine the profit of the others.

The experimenter has the opportunity and task of designing his experiments so that he will make his data as useful as possible. With respect to oligopoly experiments, the problem is to allow certain relevant factors to vary so that their effects may be measured and to keep irrelevant factors, and factors which the experimenter does not wish to study at the moment, constant. The factors which were allowed to vary in the first series of experiments were L and R. In terms of payoff per period, L was allowed to have three values, approximately 4c., 10c., and 20c. Assuming about fifteen periods could be run per hour, these three amounts would correspond to hourly rates of income of $.60, $1.50 and $3.00. It was hoped that these rates would seem to the subjects to be extremely low, average, and very high. (I.e., it was hoped these rates would seem very different to the subjects.)

The variable R was allowed two values, approximately .25 and .75. It was hoped that these values would result in, respectively, quite small differences in payoff, reading across the matrices, and quite large differences. n, the number of players, was 2 in all games. With only three values of L and two values of R, there are only six possible combinations. All of them were used. There were to be eight subjects participating in the first series of games, each one playing with each of the six matrices. They were to be paired with a new person in each game so that each subject would have played with six different subjects in the course of the games.

In the second series of games, the plan was to have L and R vary, and to have n take on the values 2, 3, and 4. This posed problems from two points of view. First, it seemed desirable to hold more than six games, so that subjects could play in three or four games of each size (two, three, and four person); yet, it would be undesirable to have the subjects play in more than eight or ten games because of the number of separate sessions which would be involved. The second problem was that of experimental design with respect to the three factors, L, R and n. Three values of L, two values of R, and three values of n can generate $3 \times 2 \times 3 = 18$ different games. Therefore a factorial experiment (an experiment in which all different combinations of all of the factors are present) was out of the question because it would involve too many games.[1]

---

1. On experimental design in general, see William G. Cochran and Gertrude M. Cox, Experimental Designs (New York: Wiley, 1957). Chapter 5 contains a discussion of factorial experiments.

The best solution seemed to be to have three levels of L, three levels of R, and three levels of n, arranged in a Latin square design. A Latin square is a matrix in which the rows are labelled with the various levels of one factor which appear in an experiment, the columns are labelled with the various levels of a second, and the entries of the matrix show the various levels of the third factor which correspond to any row and column (i.e., particular combinations of levels of the first two factors). The matrix has some special properties: The number of levels of each of the three factors must be the same.

TABLE 1

	Values of L		
R	4	10	20
.25	2	4	3
.75	3	2	4
1.25	4	3	2

Any level of the third factor appears exactly once in any row or column. That is to say, a given level of the third factor appears exactly once with a particular level of the first factor, and exactly once with any particular level of the second factor.[1]

Table 1 above shows the design of the second series of games. The column headings are the three values of L which were used, the row titles are the three values of R, and the body of the table gives the value of n corresponding to any combination of L and R. For example, the two person games were: an L of 4 with an R of .25, an L of 10 with an R of .75, and an L of 20 with an R of 1.25. In the second series, there were nine subjects. It turns out that it is not possible for nine people to play in nine games in such a way that each person plays exactly once in each of the nine games represented by Table 1. It is possible, however, for eight of the nine people to play in all nine of the games, while the ninth plays in each of the duopolies twice and plays in one of the four person games.

The actual payoff matrices were derived by making a simple linear transformation of one particular matrix. The model underlying the games has the following parameter values: $a_1 = 100$, $a_2 = 5.5$, $a_3 = 4.0$, $c_1 = 700$, $c_2 = 10$, and $c_3 = .1$. The resultant payoff matrix for any particular game may be had by multiplying all payoffs by a constant and adding a second constant to all payoffs. Thus if $\pi_i = P_i q_i - c_1 - c_2 q_i - c_3 q_i^2$ is the payoff corresponding to the original parameter values and

1. On Latin squares, see Cochran and Cox, op. cit., pp. 117–32.

$$\pi_i^* = \alpha\pi + \delta = P_i^* q_i^* - c_1^* - c_2^* q_i^* - c_3^* q_i^{*2}$$

is the desired payoff function, and it is required that $P_i^* = \beta P_i$, then:

$$\pi_i^* = \beta P_i\left(\frac{\alpha}{\beta} q_i\right) - (\alpha c_1 - \delta) - \beta c_2\left(\frac{\alpha}{\beta} q_i\right) - \frac{\beta^2}{\alpha} c_3\left(\frac{\alpha^2}{\beta^2} q_i^2\right)$$

Hence, $c_1^* = \alpha c_1$, $c_2^* = \beta c_2$, and $c_3^* = \frac{\beta^2}{\alpha} c_3$.

$$q_i^* = \frac{\alpha}{\beta} q_i = \frac{\alpha}{\beta} a_1 - \frac{\alpha}{\beta^2} a_2 P_i + \frac{\alpha}{\beta^2} a_3 \bar{P}$$

and

$$a_1^* = \frac{\alpha}{\beta} a_1,\ a_2^* = \frac{\alpha}{\beta^2} a_2 \text{ and } a_3^* = \frac{\alpha}{\beta^2} a_3.$$

IV. The First Series of Games

There were six games in the first series. Table 2 gives the value of the coefficients used in generating the payoff matrices, and the noncooperative equilibrium and joint maximum prices (P_{nco} and P_{jm}). The six games were played in the order in which they appear in Tables 2 and 3; however, this order was randomly chosen. Table 3 shows the actual values of L and R which correspond to each of the six games of the series.

TABLE 2

Game	a_1	a_2	a_3	c_1	c_2	c_3	P_{nco}	P_{jm}
1	9.09	.33	.12	107.58	15.0	1.65	51	63
2	.167	.00153	.000556	5.26	60.0	360.0	203	243
3	9.09	.66	.24	53.84	7.5	.825	25.5	31.5
4	.545	.00667	.00242	12.77	45.0	82.50	152	185
5	1.212	.0176	.0064	22.5	37.5	30.94	128	158
6	.809	.0165	.006	21.52	30.0	33.0	102	126

The original player pairings, which were so arranged that no subject was paired with the same person twice, could not be carried out, because one of the subjects, Player B, did not show up on Saturday to play games 3 through 6. As a result, alternate player pairings had to be used. The pairings which were actually used appear in Table 4, below. Player C was paired with a pre-selected dummy strategy for the last four games and it was necessary to repeat two previously used player pairings involving E, F, G, and H. No parameters were estimated for Player B because of insufficient

TABLE 3

Game	L	R
1	21.0	.7762
2	4.1	.2683
3	10.5	.7714
4	9.9	.2727
5	20.5	.2585
6	4.2	.7857

data, nor were parameters estimated for Player C because the four last games he played violated the conditions under which the subjects were to play—he was not paired with fellow subjects. For Players E, F, G, and H, parameters were estimated using data taken from all games each was in preceding the game which involved a repeated pairing for a player. Thus, data for E and H come from games 1–4, and data for F and G come from games 1–5. Only Players A and D have parameters estimated from data drawn from the full set of six games. The data taken from each game (in both series) were the ten periods immediately preceding the last period. It was decided (prior to running the games) that these periods would be used. The reason for taking the same number from each game was so that each game used would contribute equal weight to the regression analysis. Ten periods were used because the behavior which was to be used and taken as typical was the behavior of subjects after they had gotten used to the game and to the other player(s) in the game. The point estimates of the parameters for each player appear below in Table 5, along with the standard errors (appearing in parentheses beneath the coefficients) and Student "t" statistics (appearing below the standard errors) for each coefficient and the F ratio for each equation. The last columns of the table contain the critical F values for 5 per cent and 1 per cent significance levels for each equation.

TABLE 4

Game	Player Pairing			
1	AB	CD	EF	GH
2	AC	BD	EG	FH
3	AG	DF	EH	C[1]
4	AE	DH	FG	C[1]
5	AH	DE	FG[2]	C[1]
6	AF[3]	DG[3]	EH[2]	C[1]

1. Due to the absence of Player B, Player C had dummy players as competitors in these games.
2. These pairings appear for the second time; hence data from them are not used in the analysis.
3. These games are used to provide data only for Players A and D.

TABLE 5[1]

Player	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	$\hat{\beta}_5$	$\hat{\beta}_6$	F	$F_{5\%}$	$F_{1\%}$
	-.87	-.014	2.25	2.09	-.0038	-2.68	12.85	2.25	3.15
A	(1.23)	(.024)	(1.47)	(1.41)	(.034)	(1.75)			
	-.71	-.56	1.53	1.48	-.11	-1.54			
	.045	-.018	.55	.60	.0014	-.66	2.32	2.27	3.15
D	(.32)	(.02)	(.49)	(.36)	(.02)	(.54)			
	.14	-.95	1.13	1.68	.07	-1.21			
	.38	-.019	.11	.63	.026	-.44	39.05	2.31	3.24
E	(.43)	(.02)	(.31)	(.49)	(.02)	(.44)			
	.90	-1.10	.36	1.29	1.19	-1.01			
	.38	-.067	1.26	.23	.12	-2.20	9.91	2.38	3.38
F	(.33)	(.03)	(1.07)	(.31)	(.04)	(1.11)			
	1.15	-2.04*	1.18	.73	2.68**	-1.98*			
	-5.89	-.021	8.26	8.06	-.027	-10.08	35.72	2.38	3.38
G	(9.57)	(.015)	(12.5)	(10.8)	(.08)	(13.0)			
	- .62	-1.44	.66	.75	-.35	-.77			
	.57	-.0045	-.46	.70	-.0012	-.56	6.58	2.31	3.24
H	(.18)	(.014)	(.34)	(.30)	(.02)	(.45)			
	3.14**	-.34	-1.34	2.36*	-.06	-1.23			

* Significantly different from zero at the 95 per cent level.
** Significantly different from zero at the 99 per cent level.

1. If any coefficients, divided by their standard errors, do not equal their t's, the discrepancy is due to rounding.

Looking at the six equations separately, we see from Table 5 that all the F values are significant at the 1 per cent level, except for the F for Player D. That value, 2.32, is significant at the 5 per cent level. The alternate hypothesis which those F ratios represent is the hypothesis that all six coefficients equal zero. In addition to testing the hypothesis that all coefficients are zero, seven additional alternate hypotheses were chosen to be tested. They are:

1. $\beta_1 = \beta_2 = \beta_3 = 0$	which means that $\alpha_1 = 0$
2. $\beta_4 = \beta_5 = \beta_6 = 0$	which means that $\alpha_2 = 0$
3. $\beta_2 = \beta_3 = \beta_5 = \beta_6 = 0$	which means that neither L nor R affect the player's decisions
4. $\beta_2 = \beta_5 = 0$	which means that L does not affect the player's decisions
5. $\beta_3 = \beta_6 = 0$	which means that R does not affect the player's decisions
6. $\beta_2 = \beta_6 = 0$	which means that L and R do not affect α_1
7. $\beta_5 = \beta_6 = 0$	which means that L and R do not affect α_2

In Table 6, below, is presented one of these alternate hypotheses for each player. The alternate chosen for a player had to fulfill three criteria: First, the alternate had to be not significantly different from the null hypothesis (i.e., the full equation, presented in Table 5) at the 95 per cent level. Second, among all alternate hypotheses satisfying the first criterion, the ones were chosen for which the greatest number of coefficients were restricted to zero. Third, among all hypotheses satisfying both the preceding criteria, the one was chosen which had the smallest F ratio (i.e., the one which was least different from the null hypothesis). Table 6 shows the estimates of the unrestricted coefficients, given the restrictions on the others, the standard errors of the unrestricted coefficients, and the Student "t" for them.

TABLE 6[1]

Player	$\hat{\beta}_1$	β_2	$\hat{\beta}_3$	β_4	$\hat{\beta}_5$	$\hat{\beta}_6$
	.702	0	0	.052	0	0
A	(.139)	-	-	(.192)	-	-
	5.05**	-	-	.27	-	-
	.125	0	0	.211	0	0
D	(.122)	-	-	(.131)	-	-
	1.03	-	-	1.61	-	-
	.099	0	0	.885	0	0
E	(.07)	-	-	(.09)	-	-
	1.41	-	-	9.39**	-	-
	.201	0	0	.294	.062	-1.031
F	(.115)	-	-	(.210)	(.029)	(.453)
	1.74	-	-	2.42*	2.16*	-2.28*
	.365	0	0	.262	0	0
G	(.102)	-	-	(.109)	-	-
	3.58**	-	-	2.42*	-	-
	.289	0	0	.456	0	0
H	(.077)	-	-	(.110)	-	-
	3.75**	-	-	4.13**	-	-

* Significantly different from zero at the 95 per cent level.
** Significantly different from zero at the 99 per cent level.

It is, perhaps, easier to understand the point estimates of Table 5, and to some extent of Table 6, if the values of α_1 and α_2 which are implied by them are computed and examined. Table 7, below, shows the values of α_1 and α_2 which are implied by the unrestricted least squares estimates shown in Table 5. It will be recalled that:

$$\alpha_1 = \beta_1 + \beta_2 L + \beta_3 R$$

$$\alpha_2 = \beta_4 + \beta_5 L + \beta_6 R$$

Most of the entries in Table 7 fall within the expected limits of:

$$0 \leq \alpha_2 \leq 1$$

$$-(1 - \alpha_2) \leq \alpha_1 \leq (1 - \alpha_2)$$

1. If a coefficient divided by its standard error does not equal its t, the difference is due to rounding.

Some of the entries for G and A, however, are considerably outside these bounds.

For the alternate hypotheses shown in Table 6, $\alpha_1 = \beta_1$ and $\alpha_2 = \beta_4$ for all subjects except F. Table 8 gives the values for α_1 and α_2 for the four games for which data was taken for F, using the coefficients of the alternate hypothesis shown in Table 6. All values of α_1 and α_2 based on the alternate hypotheses shown in Table 6 are within the expected ranges.

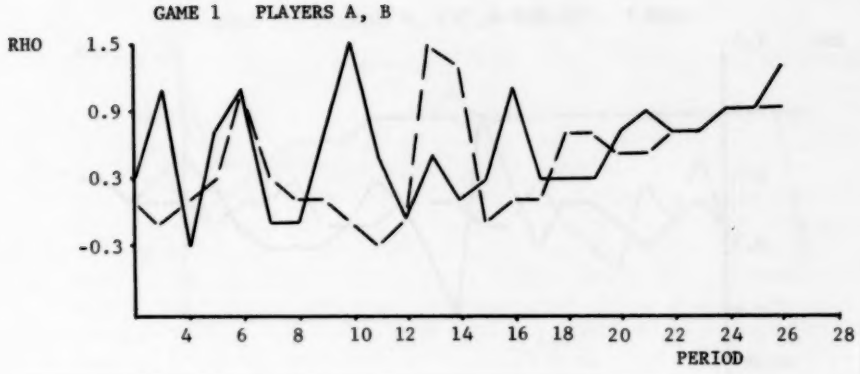
Thus far, the games have been looked at from the point of view of formal analysis, using data, models and methods of analysis which were decided upon before any examination of the data. On the next eight pages appear graphs of rho for each player in each game. Each graph represents a single game. Thus, the first graph, at the top of the next page, is for "game 1" and "Players A and B." Game 1 means the game was chronologically the first and it used the matrix denoted by "game 1." In the first game, as can be seen from Table 4, A and B were paired, as were C and D,

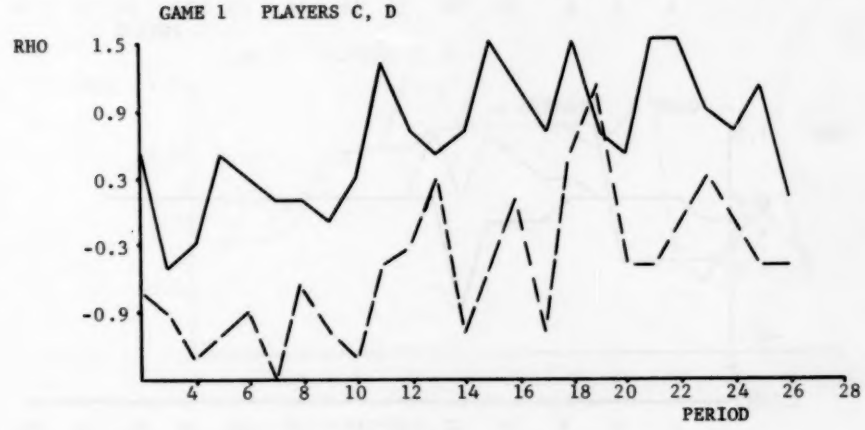
TABLE 7

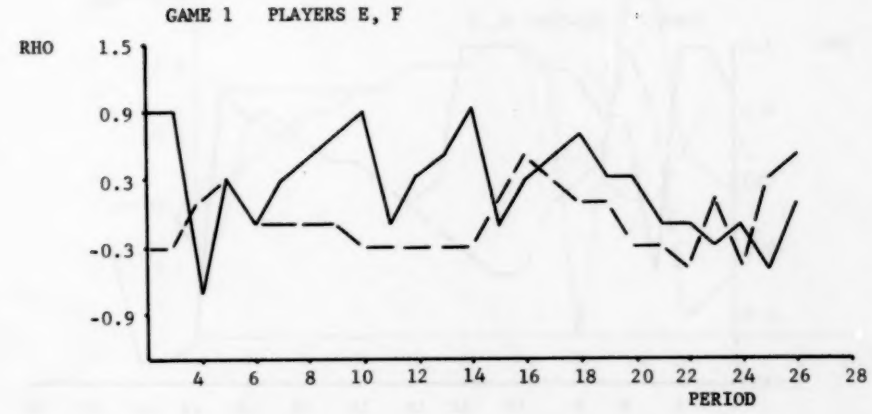
Game		Player A	D	E	F	G	H
1	α_1	.58	.09	.10	−.05	.08	.12
	α_2	−.07	.12	.84	1.04	−.33	.24
2	α_1	−.32	.12	.33	.44	−3.76	.43
	α_2	1.36	.43	.62	.13	5.25	.54
3	α_1	.72	.29	.27	.68	−.32	.17
	α_2	.01	.11	.55	−.26	.07	.26
4	α_1	−.39	.02	.22	.05	−3.95	.40
	α_2	1.32	.43	.77	.83	5.20	.54
5	α_1	−.57	.08	.03			.28
	α_2	1.31	.46	1.04			.53
6	α_1	.85	.40				
	α_2	.05	.09				

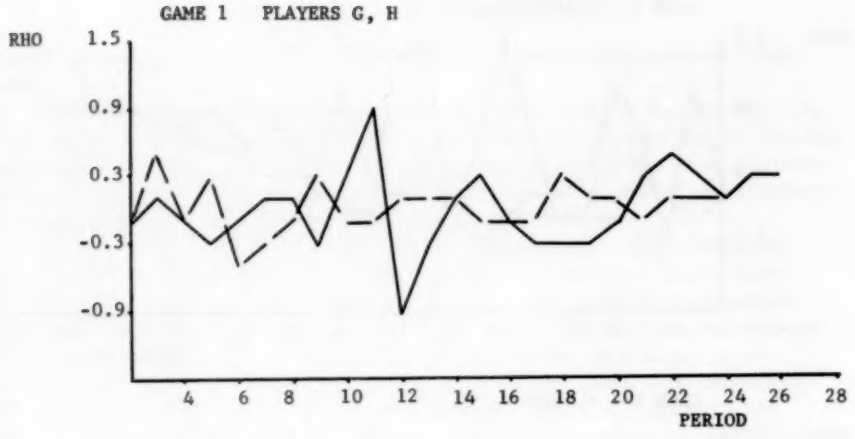
TABLE 8

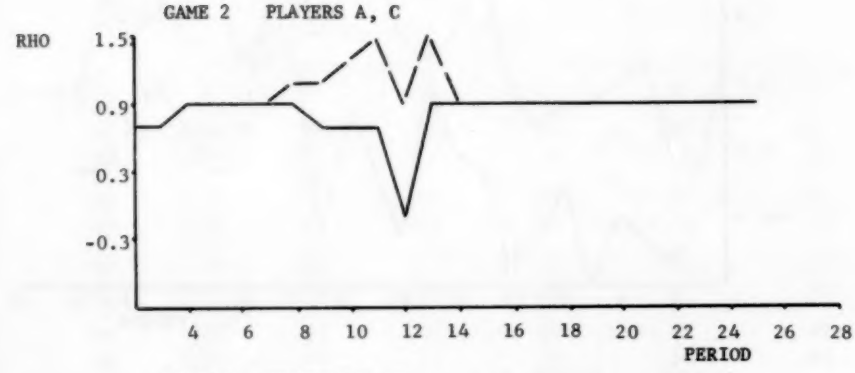
Game		Player F
1	α_1	.201
	α_2	.796
2	α_1	.201
	α_2	.272
3	α_1	.201
	α_2	.150
4	α_1	.201
	α_2	.627

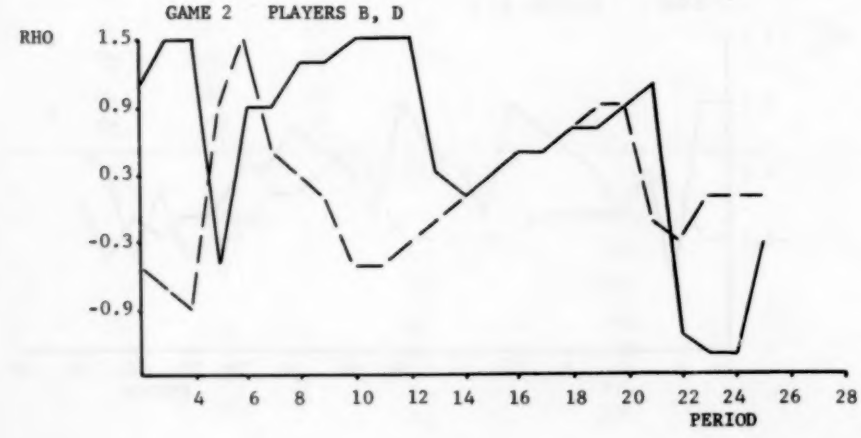

GAME 1 PLAYERS A, B
RHO
1.5
0.9
0.3
-0.3
4 6 8 10 12 14 16 18 20 22 24 26 28
PERIOD

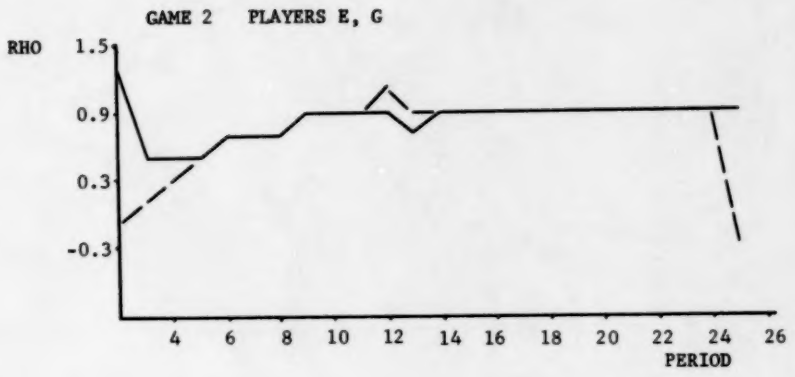

GAME 1 PLAYERS C, D
RHO
1.5
0.9
0.3
-0.3
-0.9
4 6 8 10 12 14 16 18 20 22 24 26 28
PERIOD

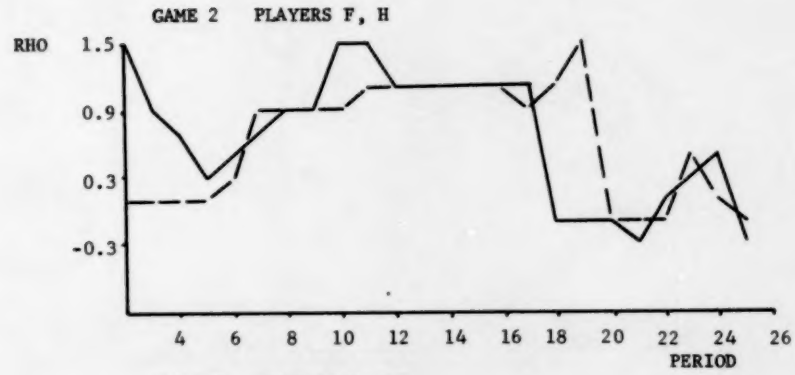

GAME 1 PLAYERS E, F
RHO
1.5
0.9
0.3
-0.3
-0.9
4 6 8 10 12 14 16 18 20 22 24 26 28
PERIOD

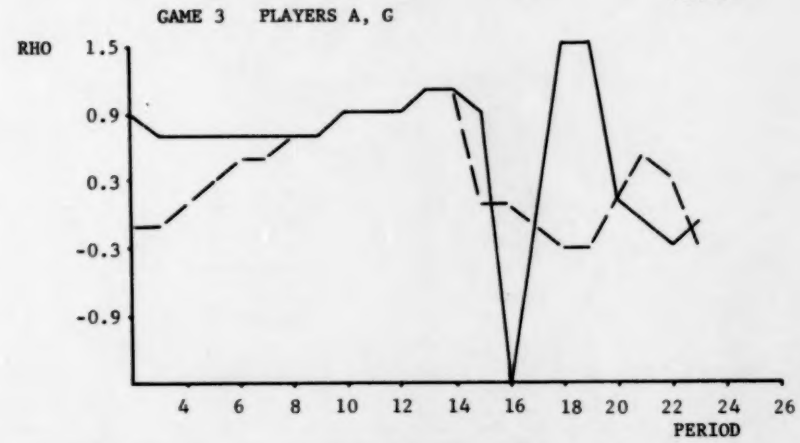

GAME 1 PLAYERS G, H
RHO
1.5
0.9
0.3
-0.3
-0.9
4 6 8 10 12 14 16 18 20 22 24 26 28
PERIOD

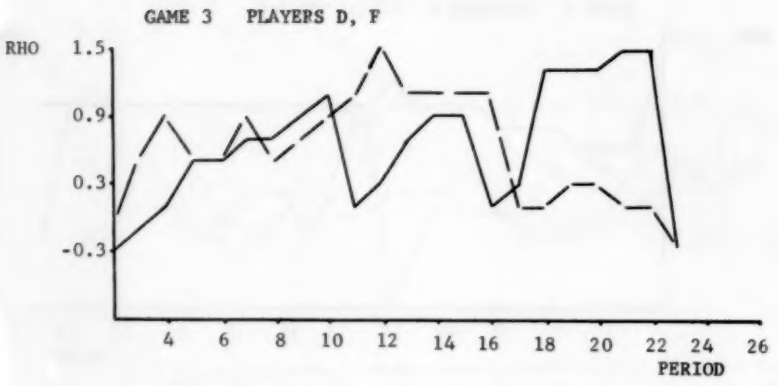

GAME 2 PLAYERS A, C
RHO
1.5
0.9
0.3
-0.3
4 6 8 10 12 14 16 18 20 22 24 26 28
PERIOD

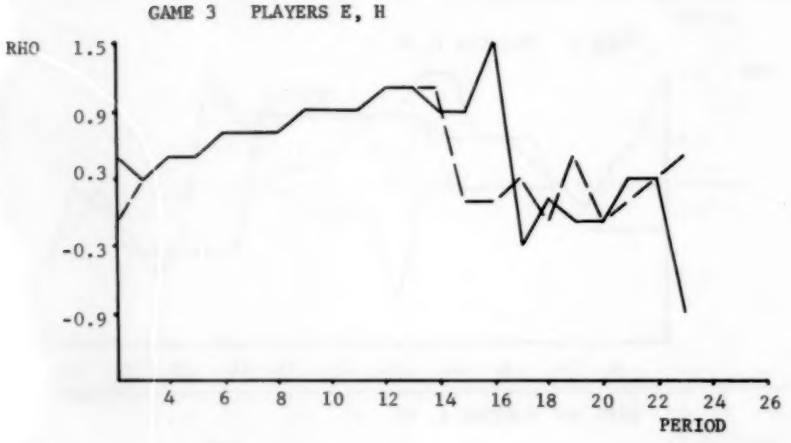

GAME 2 PLAYERS B, D
RHO
1.5
0.9
0.3
-0.3
-0.9
4 6 8 10 12 14 16 18 20 22 24 26 28
PERIOD

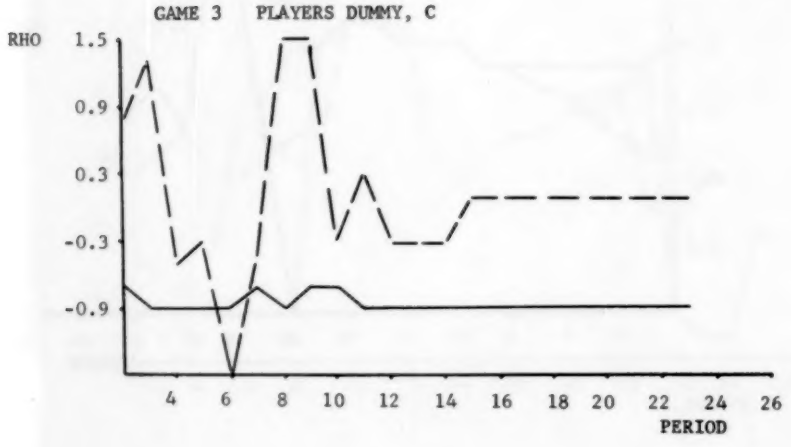

GAME 2 PLAYERS E, G
RHO
1.5
0.9
0.3
-0.3
4 6 8 10 12 14 16 18 20 22 24 26
PERIOD

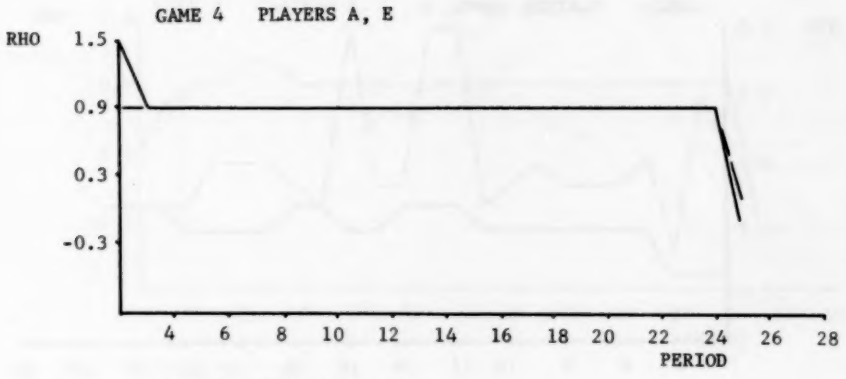

GAME 2 PLAYERS F, H
RHO
1.5
0.9
0.3
-0.3
4 6 8 10 12 14 16 18 20 22 24 26
PERIOD

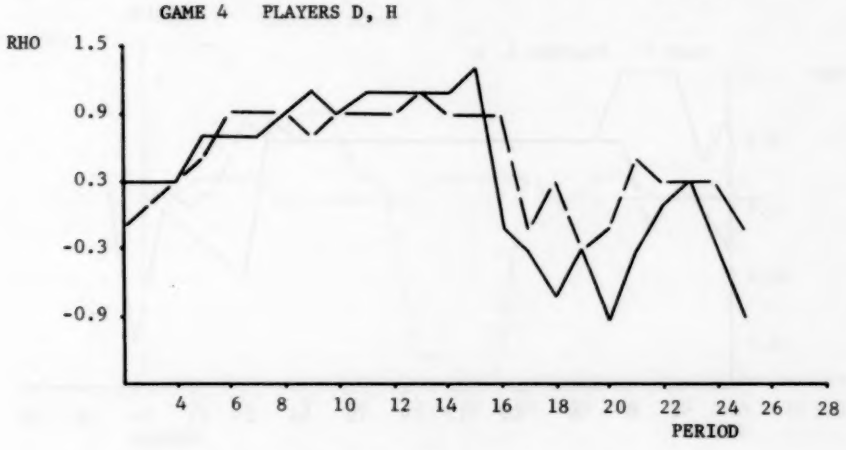

GAME 3 PLAYERS A, G
RHO
1.5
0.9
0.3
-0.3
-0.9
4 6 8 10 12 14 16 18 20 22 24 26
PERIOD

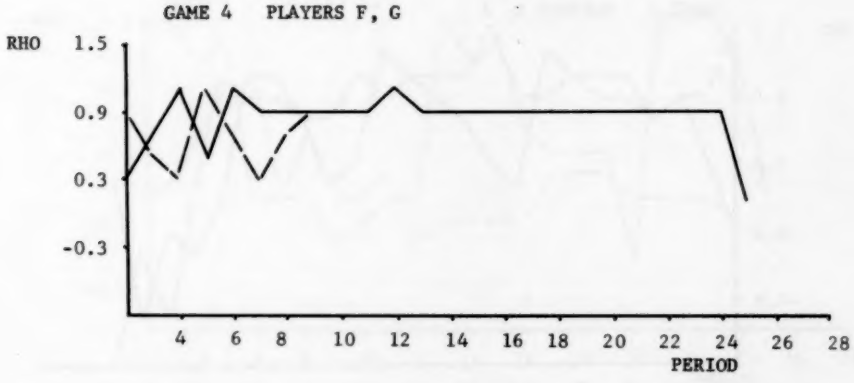

GAME 3 PLAYERS D, F
RHO
1.5
0.9
0.3
-0.3
4 6 8 10 12 14 16 18 20 22 24 26
PERIOD

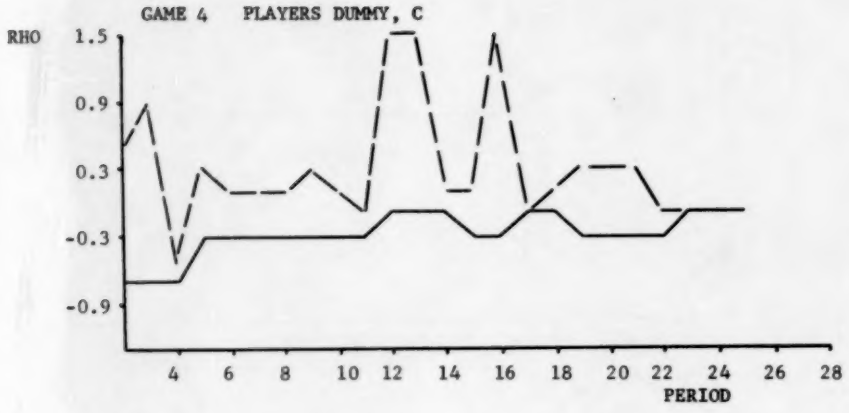

GAME 3 PLAYERS E, H
RHO
1.5
0.9
0.3
-0.3
-0.9
4 6 8 10 12 14 16 18 20 22 24 26
PERIOD

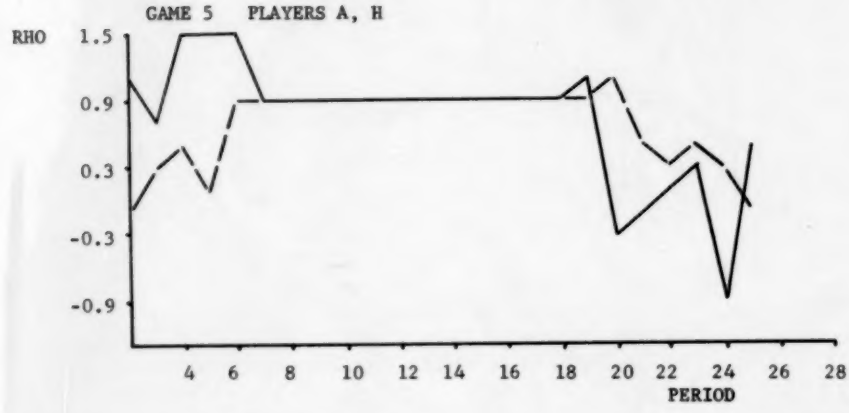

GAME 3 PLAYERS DUMMY, C
RHO
1.5
0.9
0.3
-0.3
-0.9
4 6 8 10 12 14 16 18 20 22 24 26
PERIOD

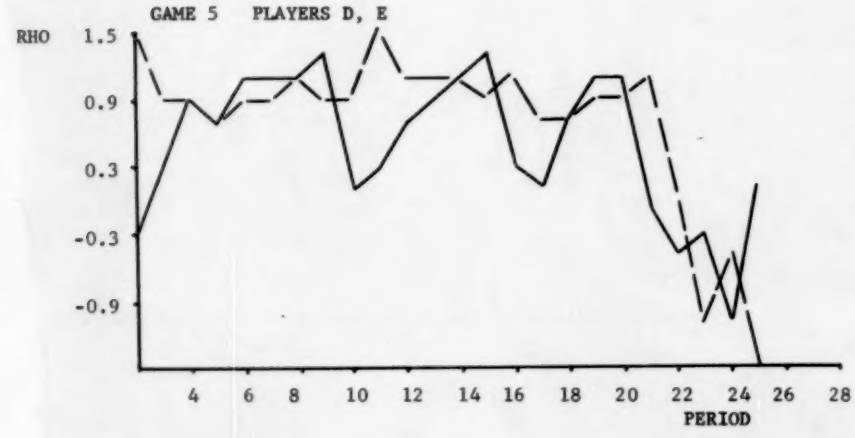

GAME 4 PLAYERS A, E
RHO
1.5
0.9
0.3
-0.3
4 6 8 10 12 14 16 18 20 22 24 26 28
PERIOD

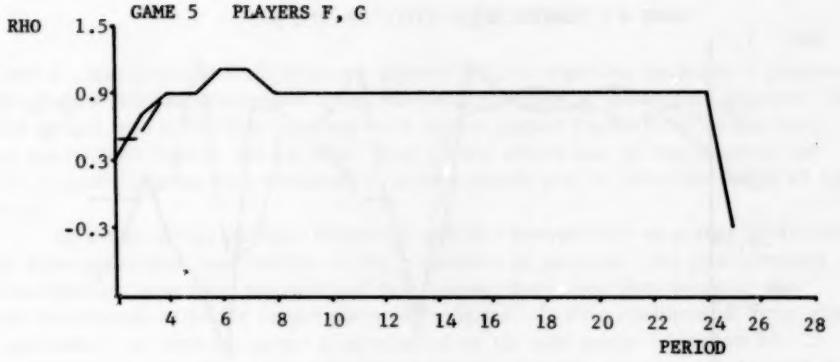

GAME 4 PLAYERS D, H
RHO
1.5
0.9
0.3
-0.3
-0.9
4 6 8 10 12 14 16 18 20 22 24 26 28
PERIOD

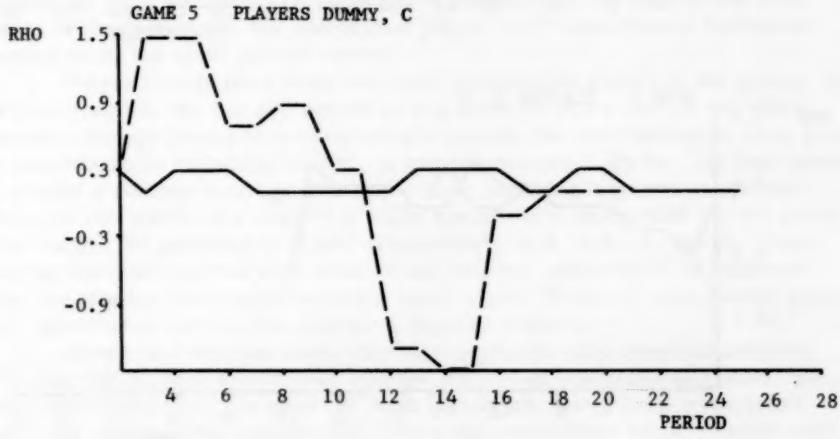

GAME 4 PLAYERS F, G
RHO
1.5
0.9
0.3
-0.3
4 6 8 10 12 14 16 18 20 22 24 26 28
PERIOD

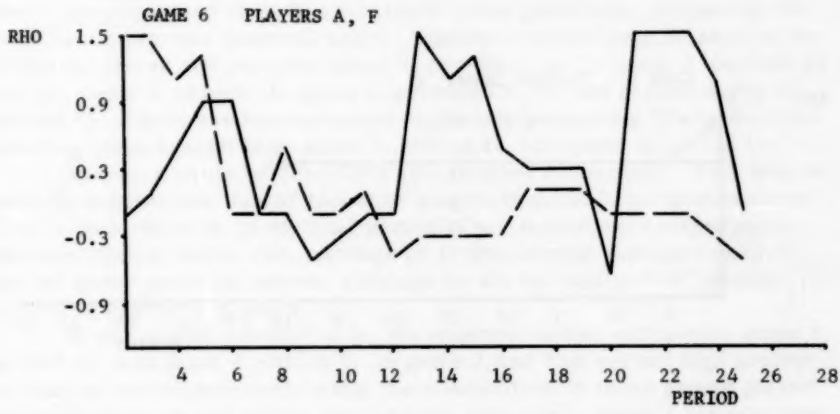

GAME 4 PLAYERS DUMMY, C
RHO
1.5
0.9
0.3
-0.3
-0.9
4 6 8 10 12 14 16 18 20 22 24 26 28
PERIOD

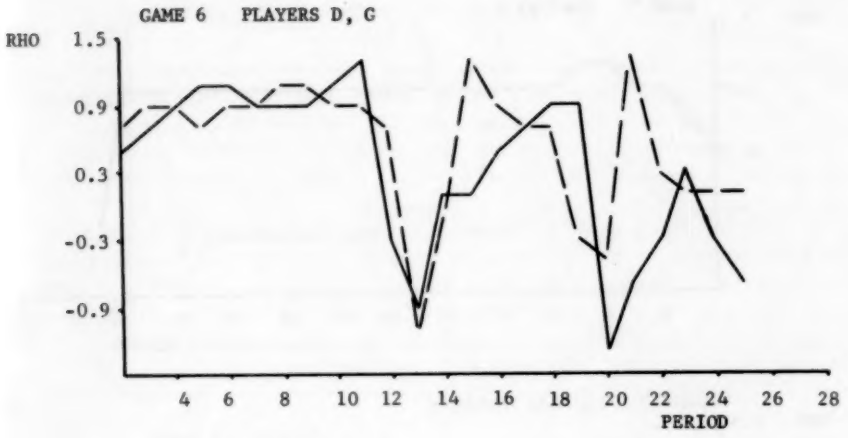

GAME 5 PLAYERS A, H
RHO
1.5
0.9
0.3
-0.3
-0.9
4 6 8 10 12 14 16 18 20 22 24 26 28
PERIOD

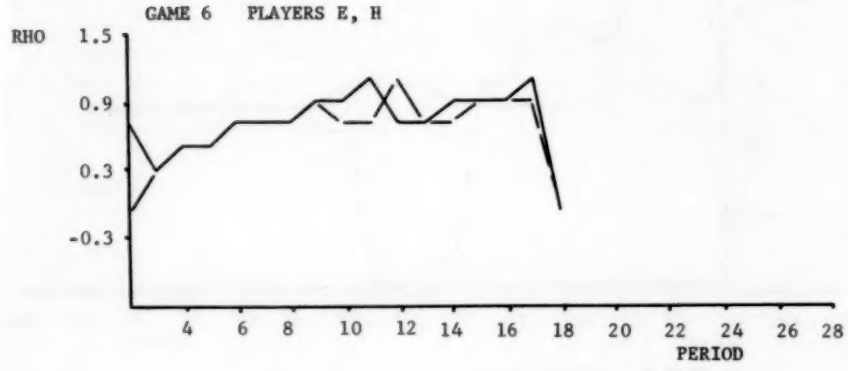

GAME 5 PLAYERS D, E
RHO
1.5
0.9
0.3
-0.3
-0.9
4 6 8 10 12 14 16 18 20 22 24 26 28
PERIOD

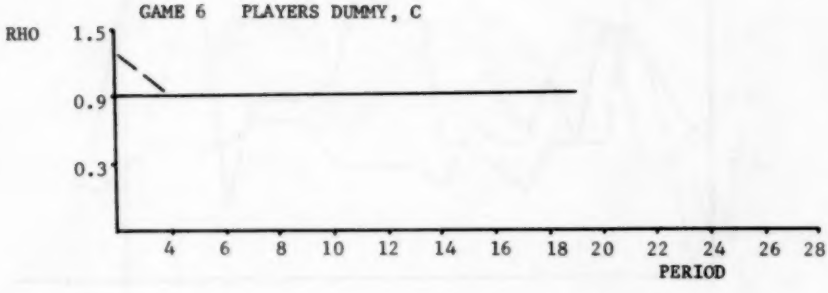

GAME 5 PLAYERS F, G
RHO
1.5
0.9
0.3
-0.3
4 6 8 10 12 14 16 18 20 22 24 26 28
PERIOD


GAME 5 PLAYERS DUMMY, C
RHO
1.5
0.9
0.3
-0.3
-0.9
4 6 8 10 12 14 16 18 20 22 24 26 28
PERIOD


GAME 6 PLAYERS A, F
RHO
1.5
0.9
0.3
-0.3
-0.9
4 6 8 10 12 14 16 18 20 22 24 26 28
PERIOD


GAME 6 PLAYERS D, C
RHO
1.5
0.9
0.3
-0.3
-0.9
4 6 8 10 12 14 16 18 20 22 24 26 28
PERIOD


GAME 6 PLAYERS E, H
RHO
1.5
0.9
0.3
-0.3
4 6 8 10 12 14 16 18 20 22 24 26 28
PERIOD


GAME 6 PLAYERS DUMMY, C
RHO
1.5
0.9
0.3
4 6 8 10 12 14 16 18 20 22 24 26 28
PERIOD

E and F, and G and H—all of whom appear paired together on game 1 graphs. The graphs will be discussed from the point of view of individual players. On each graph, the solid line corresponds to the player listed first at the top, and the broken line to the second. Any points which fall on the topmost or bottom positions may correspond to points which are outside the range of the group.

In three of six games, Player A and his competitor was able to attain the joint maximum and hold to it for a number of periods. He was tempted occasionally, however, to undercut his competitor when they were at the joint maximum in order to get very high points. In this connection, note game 1, periods 7, 8, and 14; game 2, periods 8 to 12; and game 5, period 20. In game 3, his competitor undercut him first (period 16). In game 4, he and E played the joint maximum from the third period until the next to the last; while in the last game, his competitor played very consistently noncooperatively from the sixth period onward.

Player C may have been the most cooperative player of the group. It is rare that his rho for any period of any game is below that of the other player. Except for an occasional single period, the only exception is in game 5, periods 12 to 21, when, said C, "I lost my temper." In the last four games C played a dummy strategy which, for each period of a game, chose that value of rho which was nearest a value specifically designated for the game. The values for games 3 to 6 are, respectively, −.8, −.2, .2, and .9. Comparing the dummy rhos with those of any regular subjects, it is apparent that the dummy strategies were far more rigid. Player C was always given the impression that he was playing a regular subject.

D was in a way the most interesting player. His apparent strategy was one which does not fit well into the pattern of analysis used here. He consistently tried to get near the joint maximum, sometimes even above, and then undercut his competitor. When the competitor would considerably lower his price (and rho), D would try to show good faith, get back to the joint maximum, and undercut again. The undercutting may be seen in the following games and periods: game 2, periods 7 to 11; game 3, periods 11 and 16; game 4, period 16; game 5, periods 10, 16, and 21; and game 6, period 12. There are two instances of players perceiving D's tactics and adopting them against him; game 2, period 21, and game 6, period 19.

Player E could be described as a passive cooperator. This may be seen by considering that he had to be taught, in game 2, to "double cross" (i.e., play a rho of 0) in the final period of a cooperatively played game. He submitted to more undercuttings by D than anyone else (see game 5) and he never undercut anyone, although he did the last period "double cross" in games 3, 4, and 6.

F was a mild version of D. He undercut on two occasions: game 2, period 18, and game 4, period 7. In game 1 and 6 he earned high profits by playing noncooperatively while his competitors in those games played cooperatively. G was fairly cooperative, although in game 1 his play was predominately noncooperative.

H played curiously. His first game was played basically noncooperatively; however, in the second game, which was played with F, he was undercut in period 16. Then he seemed to react in the next game (3) by undercutting his competitor in period 16. In the fourth game D undercut him in period 16, and then he reacted in game 5 by undercutting A in period 20. All subjects were given a questionnaire after all games were over. Some of the answers given by H are interesting:

Q: What was your objective in the project?

A: To earn as much as possible.

Q: What strategy did you follow to achieve this objective?

A: [I] indicated that I would go up with him to the optimum level for us both. [I] tried to show him I was not out to give him losses but that together we could make money.

Q: What were the apparent strategies of the other sellers?

A: . . . probably the same . . .

Q: What were their apparent strategies?

A: Luckily on some of the bigger games they had similar motives to mine and we stayed at the optimum for many moves. Sometimes either of us would take a loss when this became too boring.

Judging especially by the last answer, it would seem the money at stake was not always enough to make H single-minded about profits.

The analysis for "end effects" is based on the hypothesis that subjects behave differently when the end of the game is known to be imminent than when the game may go on for an indefinite time. Data was generated by the practice of telling the subjects just before handing back results for the next to last period that the next period for which a decision was due would be the last period of the game. In Table 9, below, appear the number of rho values for the very last period of the games which are the approximate sizes shown across the top of the table. Thus, there were 15 last period rho values which were less than $-.2$; 8 which were between $-.2$ and 0; etc. All players in all games were used, except for the dummy strategy in the last four games. Thus, there are 44 observations. The numbers in parentheses below these frequencies are the expected frequencies. The expected frequencies are computed from the frequency of rho values in each of the five categories in the five periods preceding the last period for each game from which a last period rho value was taken.

TABLE 9

	< −.2	−.2 to 0	0 to .2	.2 to .6	> .6
Observed Frequencies	15	8	10	5	6
Expected Frequencies	(6.4)	(5.2)	(6.2)	(6.6)	(19.6)

For example, 32 of the rho values for the five periods preceding the last one were less than −.2. That is 32 out of 220. If final period play does not differ, in general, from the play of the immediately preceding periods, we would expect the same fraction of last period rho values to be less than −.2. This fraction is 32/220 which equals 6.4/44, thus 6.4 is the expected frequency of observations in the first cell of the table.

The hypothesis may be tested by a chi-square test, with 4 degrees of freedom. If the distribution of actual last period frequencies does not differ significantly from the expected frequencies, the χ^2 value computed from the data in the table will be less than the critical value from a χ^2 table. The value computed from the table above is 25.23, which exceeds the value from the χ^2 table (for a 99 per cent significance level). The critical value is 13.28. Thus we must accept the hypothesis that last period play differs from play in the immediately preceding periods.

V. The Second Series of Games

The coefficients used to generate the nine payoff matrices which were employed in the second series of games appear in Table 10, along with the noncooperative equilibrium price and the joint maximum price of each game. Each game is identified by a letter and a number. The number tells how many firms are in the industry using the particular matrix and the letter tells to which "group" the game belongs. The significance of the "group" is explained below. Table 11, gives the actual values of L and R corresponding to each of the nine distinct games. Table 12 gives the player pairings for the whole series. The number under the heading "game" gives the chronological order of the games. Thus, in the first game played by the nine subjects, A, B, C, and D played in the four person game of the "A" group, E, H, and I played in the three person game of the "A" group, and F and G played in the two person game of the "A" group. From Table 12 it is apparent that each of the groups, A, B, and C, contain one two person game, one three person game, and one four person game, and that the first three games in which the subjects play are A games, the second three, B games, and the last three are C games. The Players

TABLE 10

Game	a_1	a_2	a_3	c_1	c_2	c_3	P_{nco}	P_{jm}
2A	21.7	3.99	1.45	50.1	3.0	.138	10.1	12.5
3A	.855	.0125	.00304	8.56	37.5	43.8	122.0	157.0
4A	3.86	.283	.0514	21.3	7.5	1.95	23.9	30.9
2B	9.96	.236	.0858	185.5	22.5	2.33	77.0	95.0
3B	1.28	.0470	.0114	13.1	15.0	11.7	49.0	62.0
4B	.481	.0088	.0016	2.63	30.0	62.5	97.0	123.0
2C	1.45	.133	.0484	4.1	6.0	4.13	20.3	23.9
3C	1.78	.0218	.00529	60.0	45.0	25.3	145.0	187.0
4C	57.8	21.2	3.85	55.8	1.5	.026	4.8	6.3

TABLE 11

Game	L	R
2A	10.0	.760
3A	20.3	.261
4A	3.8	1.34
2B	19.2	1.19
3B	4.1	.708
4B	10.1	.237
2C	4.0	.250
3C	10.6	1.250
4C	19.7	.752

TABLE 12

Game						Group
1	ABCD	EHI	FG			A
2	EFGH	ACD	BI			A
3		BFG	CD	AE	HI	A
4	ABEG	DFI	CH			B
5	CDFH	ABE	GI			B
6		CGH	AB	DI	EF	B
7	ACFG	BDH	EI			C
8	BDEH	AGI	CF			C
9		CEF	AI	BD	GH	C

A through H all play exactly once in each of the three games of a group; hence, after the third, sixth, and ninth games had been finished, those eight players had played exactly once using each of the same three, six, and nine matrices. Table 13 shows which values of L, R, and n correspond to each of the three groups. The members of each group were chosen in such a way that each value of L and each value of R appear exactly once

in each group. This, along with the requirement that each value of n appear exactly once, meant that the series was laid out in a Graeco-Latin square design with respect to the four factors, L, R, n, and group.

TABLE 13

	Values of L		
R	4	10	20
.25	2C	4B	3A
.75	3B	2A	4C
1.25	4A	3C	2B

As in the preceding section, the parameters of a regression function were estimated for players. In the second series, however, all subjects were present for all games; hence, there are ten observations from each of nine games for Players A through H. The regression function is:

$$\rho_{it} = \beta_1 + \beta_2 L_{it} + \beta_3 R_{it} + \beta_4 D_{1it} + \beta_5 D_{2it} + (\beta_6 + \beta_7 L_i + \beta_8 R_{it} + \beta_9 D_{1it} + \beta_{10} D_{2it}) \frac{1}{nt} \sum_{j \neq i} \rho_{jt-1}$$

where the ρ_{jt-1} are summed over the players in the current game with Player i (excluding Player i), and n_t is one less than the number of players in the current game with Player I. D_{1it} equals 1 when the number of players in the game is 3, and otherwise equals zero. D_{2it} equals 1 when the number of players in the game is 4, and otherwise zero. The unrestricted least squares estimates of the parameters for each of the eight players appear in Table 14.

Examining the eight regressions individually, we see that the coefficient matrix of each is significantly different from the zero matrix at the 99 percent level of significance; for all F ratios are in excess of $F(10, 80) = 2.74$, the critical value.

In addition to testing the alternate hypothesis that all coefficients equal zero, the following eight alternate hypotheses were tested:

1) $\beta_1 = \beta_2 = \beta_3 = \beta_4 = \beta_5 = 0$ i.e., $\alpha_1 = 0_1$

2) $\beta_6 = \beta_7 = \beta_8 = \beta_9 = \beta_{10} = 0$ i.e., $\alpha_2 = 0_1$

3) $\beta_4 = \beta_5 = \beta_9 = \beta_{10} = 0$ i.e., the number of players in the game has no effect on α_1 or α_2.

4) $\beta_2 = \beta_3 = \beta_7 = \beta_8 = 0$ i.e., neither L nor R affect α_1 or α_2

5) $\beta_2 = \beta_7 = 0$ i.e., L affects neither α_1 nor α_2.

6) $\beta_3 = \beta_8 = 0$ i.e., R affects neither α_1 nor α_2.

7) $\beta_2 = \beta_3 = \beta_4 = \beta_5 = \beta_7 = \beta_8 = \beta_9 = \beta_{10} = 0$ i.e., α_1 and α_2 are independent of L, R, and n.

8) $\beta_2 = \beta_3 = \beta_4 = \beta_5 = \beta_6 = \beta_7 = \beta_8 = \beta_9 = \beta_{10} = 0$ i.e., α_1 is independent of the structure of the game (L and R), the number players in the game and the behavior of the other players (as registered by their rho's).

Table 15 presents one alternative hypothesis for each player except A. The alternative hypothesis was selected by the same criteria used to select an alternate in Section IV. No alternate is selected for Player A because none met the criteria.

Table 16 presents the values of α_1 and α_2 which are implied by the coefficients in Table 14, and Table 17 presents the α_1 and α_2 values implied by the coefficients in Table 15.

Looking at Table 15, we see that the $\hat{\beta}_2$ through $\hat{\beta}_5$ are more often negative than positive, indicating that α_1 tends to decrease as L, R, and n increase. That is to say, the more at stake in a game (L), the more there is to gain, relatively, as one moves across the payoff matrix (R), and the greater the number of players, the less cooperative the players tend to be. It is apparent from Table 17 that α_1 is generally greater than or equal to zero, and it may be that, as these variables increase, α_1 tends to the noncooperative level of zero. On the other hand, the $\hat{\beta}_7$ and $\hat{\beta}_8$ tend to be greater than zero (or at least equal to zero), while the $\hat{\beta}_9$ and $\hat{\beta}_{10}$ are mixed. This would indicate that, as L and R increase, α_2 tends to increase. In other words, as L and R increase, the dependence of a player's behavior on the behavior of the others in the game tends to increase. If α_2 tends to increase as L and R increase, it is not surprising that α_1 should tend to decrease, because, α_1 being generally greater than or equal to zero, a decrease in α_1 usually means (within the context of the data) a decrease in absolute value as well. The likely limits on α_1, $-(1 - \alpha_2)$ to $(1 - \alpha_2)$ are narrowed as α_2 increases; therefore, the likely limit on the absolute value of α_1 is decreased. By inspection of Tables 16 and 17, it is apparent that α_1 and α_2 usually fall within the limits which were postulated. (The limits for α_2 are 0 and 1). We may also note that if Player D is left out of account, the remarks above about the effects of L, R, and n and the limits on α_1 and α_2 appear even stronger.

TABLE 14

Player	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	$\hat{\beta}_5$	$\hat{\beta}_6$	$\hat{\beta}_7$	$\hat{\beta}_8$	$\hat{\beta}_9$	$\hat{\beta}_{10}$	F
	.835	−.014	−.278	−.088	−.222	−.036	.035	.021	.352	.267	192
A	(.16)	(.01)	(.18)	(.19)	(.21)	(.19)	(.020)	(.21)	(.20)	(.22)	
	5.20**	−1.21	−1.59	−.46	−1.06	−.19	2.38*	.10	1.75	1.19	
	.596	−.068	.841	−1.196	−.501	.305	.069	−.764	1.278	.627	107
B	(1.07)	(.03)	(.69)	(.57)	(.58)	(1.25)	(.03)	(.80)	(.61)	(.67)	
	.55	−2.34*	1.21	−2.11*	−.87	.24	2.10*	−.96	2.09*	.94	
	.308	−.023	.126	−.131	−.352	.585	.028	−.140	−.011	−.189	23
C	(.37)	(.02)	(.30)	(.26)	(.20)	(.50)	(.03)	(.46)	(.41)	(.38)	
	.83	−1.31	.42	−.50	−1.73	1.18	1.07	−.30	−.03	−.50	
	−.241	.026	.168	.099	.194	1.478	−.062	−.545	−.121	−.025	33
D	(.24)	(.01)	(.14)	(.12)	(.17)	(.34)	(.02)	(.27)	(.22)	(.31)	
	−.99	2.48*	1.24	.83	1.13	4.35**	−3.48**	−2.02*	−.55	−.08	
	1.143	−.011	−1.009	−.402	−.118	−.300	.008	1.185	.362	.166	93
E	(1.04)	(.03)	(.60)	(.61)	(.69)	(1.24)	(.04)	(.68)	(.69)	(.80)	
	1.10	−.33	−1.68	−.66	−.17	−.24	.20	1.75	.52	.21	
	1.510	−.030	.098	−1.216	−1.073	−.846	.041	.032	1.273	1.175	142
F	(.34)	(.01)	(.14)	(.29)	(.27)	(.43)	(.01)	(.18)	(.32)	(.32)	
	4.49**	−2.79**	.72	−4.22**	−4.00**	−1.97*	3.10**	.18	3.92**	3.68**	
	1.223	−.044	−.372	−.243	−.016	−.437	.066	.161	.060	−.200	55
G	(.52)	(.02)	(.40)	(.28)	(.23)	(.59)	(.02)	(.46)	(.31)	(.27)	
	2.36*	−2.71**	−.94	−.86	−.07	−.74	3.58**	.35	.19	−.75	
	1.190	.002	−.425	−.697	−.578	−.238	−.024	.366	.700	.392	28
H	(.36)	(.02)	(.22)	(.18)	(.23)	(.51)	(.03)	(.40)	(.29)	(.44)	
	3.34*	.13	−1.95	−3.93**	−2.53*	−.47	−.84	.92	2.38*	.89	

*Significantly different from zero at the 95% level.
**Significantly different from zero at the 99% level.

TABLE 15

Player	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	$\hat{\beta}_5$	$\hat{\beta}_6$	$\hat{\beta}_7$	$\hat{\beta}_8$	$\hat{\beta}_9$	$\hat{\beta}_{10}$
	.835	−.014	−.278	−.088	−.222	−.036	.035	.021	.352	.267
A	(.16)	(.01)	(.18)	(.19)	(.21)	(.19)	(.02)	(.21)	(.20)	(.22)
	5.20**	−1.21	−1.59	−.46	−1.06	−.19	2.38*	.10	1.75	1.19
	1.263	−.090	0	−.558	.075	−.353	.094	0	.612	−.041
B	(.49)	(.02)	-	(.44)	(.46)	(.55)	(.02)	-	(.47)	(.51)
	2.60**	−5.33**	-	−1.27	.16	.64	4.66**	-	1.29	−.08
	0	0	0	0	0	.953	−.002	.106	−.259	−.701
C	-	-	-	-	-	(.14)	(.01)	(.13)	(.13)	(.15)
	-	-	-	-	-	6.77**	−.24	.83	−1.92	−4.76**
	−.010	.021	.118	0	0	1.201	−.052	−.450	0	0
D	(.17)	(.01)	(.13)	-	-	(.24)	(.01)	(.26)	-	-
	−.06	2.59**	.90	-	-	4.94**	−3.88**	−1.72	-	-
	.061	0	0	0	0	.899	0	0	0	0
E	(.08)	-	-	-	-	(.10)	-	-	-	-
	.80	-	-	-	-	9.39**	-	-	-	-
	1.481	−.034	0	−1.071	−.916	−.707	.046	0	1.105	.994
F	(.27)	(.01)	-	(.27)	(.25)	(.33)	(.01)	-	(.30)	(.30)
	5.51**	−3.63**	-	−3.97**	−3.69**	−2.14*	3.87**	-	3.66**	3.34**
	1.578	−.053	−.710	0	0	−.969	.074	.603	0	0
G	(.50)	(.02)	(.36)	-	-	(.55)	(.02)	(.41)	-	-
	3.16**	−3.32**	−1.98*	-	-	−1.76	4.05**	1.46	-	-
	1.102	−.011	−.284	−.351	−.355	0	0	0	0	0
H	(.11)	(.01)	(.09)	(.09)	(.09)	-	-	-	-	-
	10.07**	−2.02*	−3.28**	−4.00	−4.04**	-	-	-	-	-

*Significantly different from zero at the 95% level.
**Significantly different from zero at the 99% level.

TABLE 16

Game		Player A	B	C	D	E	F	G	H
2A	α_1	.48	.55	.17	.15	.27	1.29	.50	.89
	α_2	.33	.51	.76	.45	.68	−.41	.35	−.29
3A	α_1	.39	−1.77	−.26	.44	.26	−.28	−.01	.43
	α_2	1.03	2.79	1.11	−.04	.53	1.27	1.01	.08
4A	α_1	.19	.96	.04	.28	−.37	.46	.54	.05
	α_2	.39	.17	.32	.49	1.48	.53	−.17	.56
2B	α_1	.23	.29	.01	.47	−.26	1.06	−.06	.73
	α_2	.66	.72	.96	−.35	1.26	−.02	1.03	−.25
3B	α_1	.49	−.28	.17	.09	−.02	.24	.54	.20
	α_2	.47	1.32	.59	.72	.93	.62	.01	.62
4B	α_1	.41	−.39	−.25	.26	.68	.16	.68	.54
	α_2	.59	1.45	.65	.70	.23	.75	.07	0
2C	α_1	.71	.53	.25	−.09	.85	1.42	.96	1.09
	α_2	.11	.39	.66	1.10	.03	−.67	−.13	−.24
3C	α_1	.38	.38	.31	.10	−.53	.39	.47	−.04
	α_2	.38	.70	.43	.61	1.55	.51	−.10	.89
4C	α_1	.38	.59	0	.13	.24	.45	.84	.30
	α_2	.31	.49	.35	.92	.77	.43	−.39	.38

TABLE 17

Game		Player A	B	C	D	E	F	G	H
2A	α_1	.48	.36	0	.29	.06	1.17	.51	.78
	α_2	.33	.59	1.01	1.02	.90	−.25	.23	0
3A	α_1	.39	−1.12	0	.45	.06	−.23	.32	.45
	α_2	1.03	2.17	.68	.26	.90	1.33	.69	0
4A	α_1	.19	1.00	0	.23	.06	.45	.41	.33
	α_2	.39	−.04	.39	1.78	.90	.46	.12	0
2B	α_1	.23	−.47	0	.53	.06	.89	−.29	.55
	α_2	.66	1.45	1.04	.74	.90	.18	1.17	0
3B	α_1	.49	.34	0	.16	.06	.28	.86	.50
	α_2	.47	.64	.76	1.31	.90	.59	−.24	0
4B	α_1	.41	.43	0	.23	.06	.25	.88	.57
	α_2	.59	.56	.26	.78	.90	.75	−.08	0
2C	α_1	.71	.90	0	.10	.06	1.37	1.19	.98
	α_2	.11	.02	.97	1.11	.90	−.52	−.52	0
3C	α_1	.38	−.25	0	.36	.06	.08	.13	.28
	α_2	.38	1.26	.81	1.21	.90	.89	.57	0
4C	α_1	.38	−.45	0	.49	.06	−.05	0	.32
	α_2	.31	1.47	.29	.51	.90	1.19	.94	0

On the following ten pages are graphs, each of a single game. For the two person games, the first player listed corresponds to the solid line and the second to the broken. In the three person games, the first player corresponds to the solid line, the second to the broken, and the third to the dashed. In the four person games, the first player corresponds to the dotted line, the second, solid, the third, broken, and the fourth, dashed.

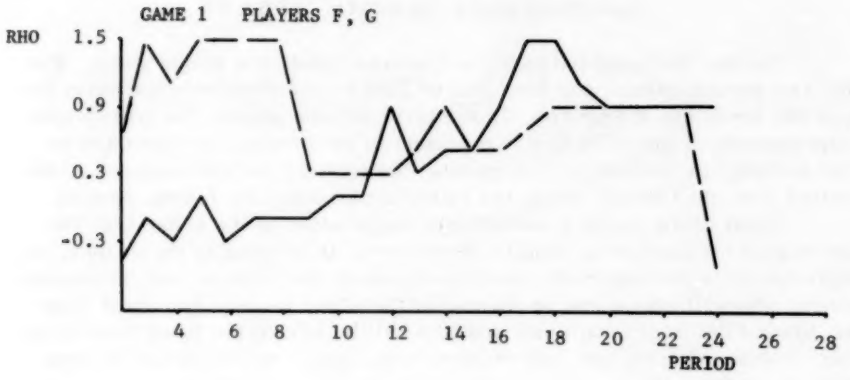
Most of the players were fairly cooperative in the sense that rho for nearly all players is usually above zero. In examining the graphs, the differences in the apparent cooperativeness of the players may be seen by noting who will play a rho above that of the other players to induce them all toward the joint maximum, and who will be among the first, middle, or last to descend from the joint maximum in a game which attains it, then falls away from it.

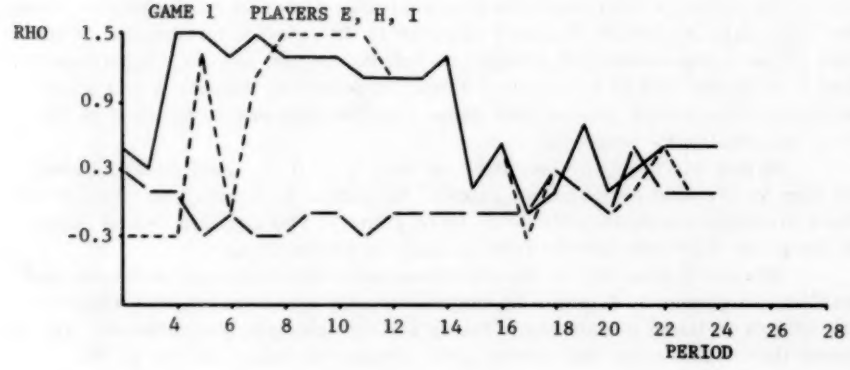
Player A was among the most cooperative of players. In games 1 and 2, he played a very high rho to induce the others to raise theirs. Note, for example, periods 5, 8, and 9 of game 1. In game 4, he was one of the two players who never left a high rho before the last period. In games 7 and 8, A is the last to break away from cooperation. Games 3 and 5 are straight cooperative games, and game 6 is the only one in which A is the first in ceasing to cooperate.

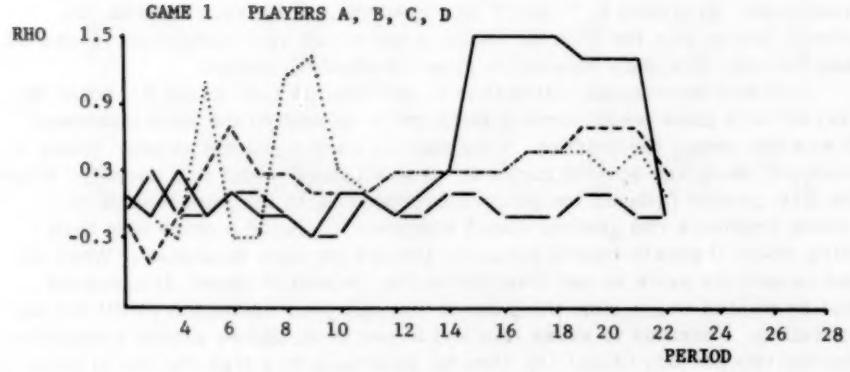
B was also very cooperative; games 2, 3, 5, 6, and 9 being played by him as straight cooperative games. In games 1, 7, and 8 he tried to induce cooperation—note periods 9–13 of game 7, and periods 7–8 of game 8. In game 4 he was not the first to give up cooperating.

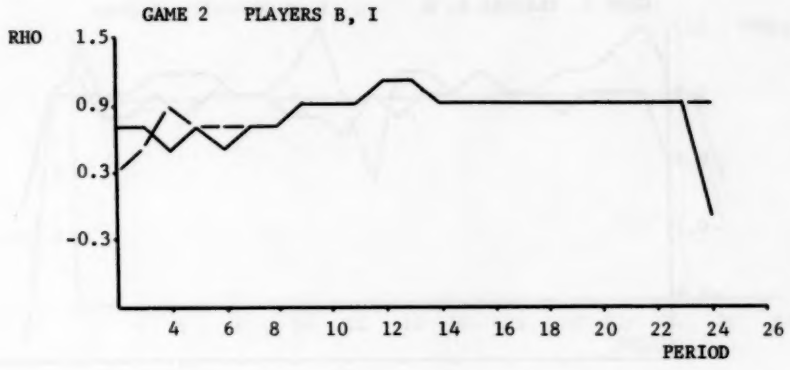
Player C was one of the most noncooperative players of the second series. In games 1, 2, and 5 he played strictly noncooperatively while the others in those games made fairly strong attempts to cooperate. In these three games his rho rarely gets outside the range of −.2 to .25. Game 3 was played like 1, 2, and 5 except that C cooperated a bit at the end, and in game 4 he was more cooperative than in the games already mentioned. In games 6, 7, and 9, C played the joint maximum with the others, but he was the first to undercut and break up cooperation. Game 8 was the only straight cooperative game in which he played.

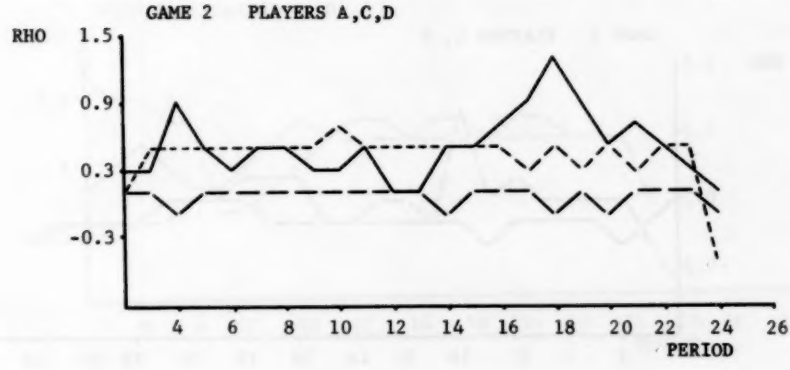
D was more cooperative than C, but less so than A and B. When the players of a game were working their rho's up toward the joint maximum, D was not among the leaders. This may be seen in all his games. Game 6, however, deserves special mention. D was paired with I in a duopoly. From the first period I played the price corresponding to the joint maximum (which implies a rho greater than I whenever D played a price less than this), while D slowly raised his price toward the joint maximum. When he had raised his price to one level below I's, he kept it there. It appeared that he wanted to get something more than the joint maximum profit for cooperating. I seemed to sense this and object to it, and he played a negative rho for two periods (9 and 10), then he went back to a high rho for several periods—as if to say, "I'm willing to cooperate if you will, but I'm not about

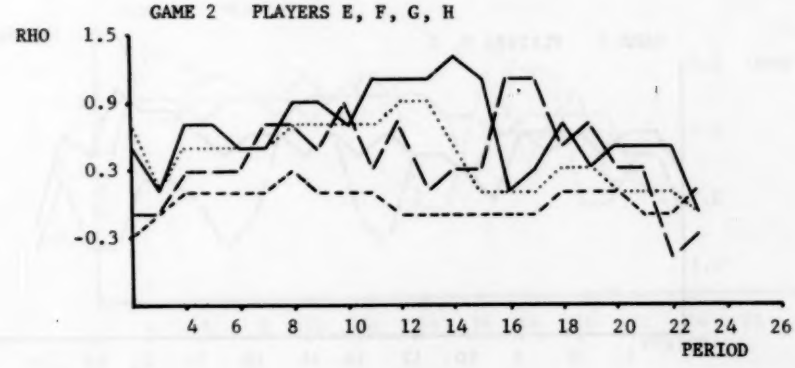

GAME 1 PLAYERS F, G
RHO
1.5
0.9
0.3
-0.3
4 6 8 10 12 14 16 18 20 22 24 26 28
PERIOD

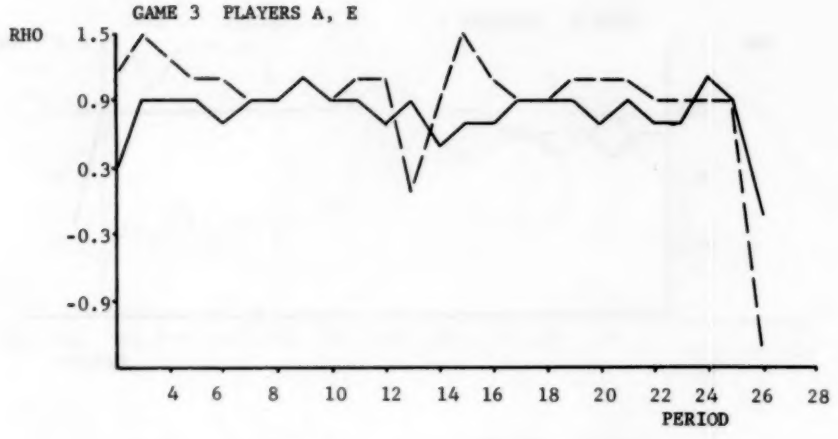

GAME 1 PLAYERS E, H, I
RHO
1.5
0.9
0.3
-0.3
4 6 8 10 12 14 16 18 20 22 24 26 28
PERIOD

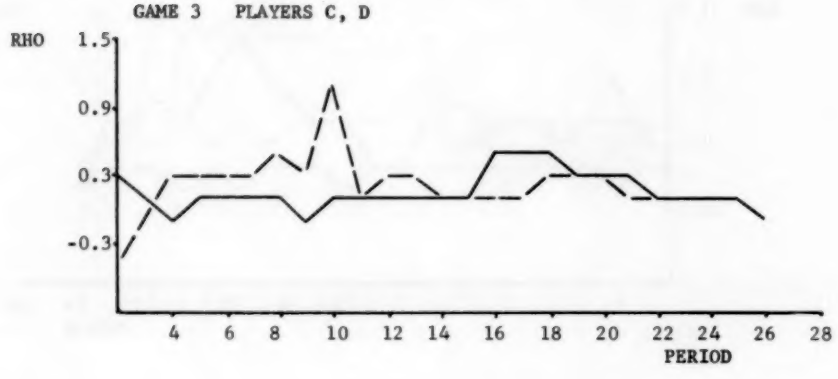

GAME 1 PLAYERS A, B, C, D
RHO
1.5
0.9
0.3
-0.3
4 6 8 10 12 14 16 18 20 22 24 26 28
PERIOD

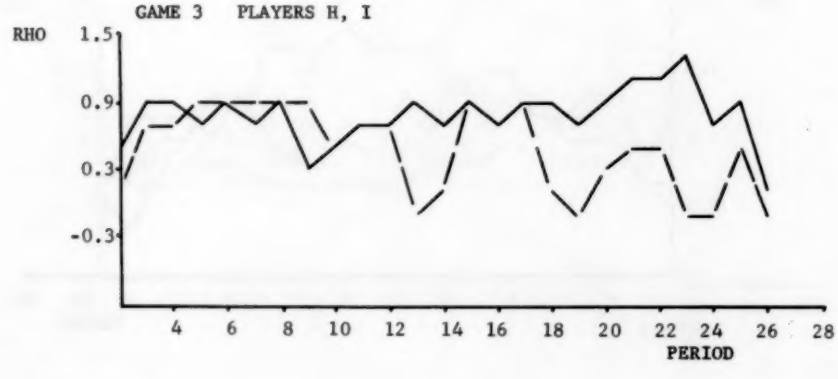

GAME 2 PLAYERS B, I
RHO
1.5
0.9
0.3
-0.3
4 6 8 10 12 14 16 18 20 22 24 26
PERIOD

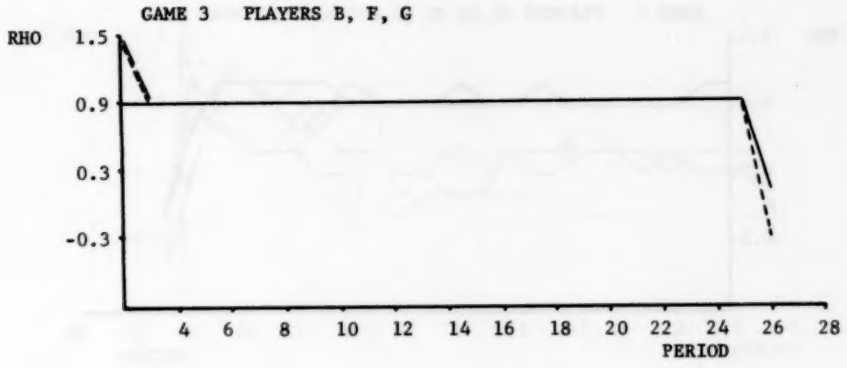

GAME 2 PLAYERS A,C,D
RHO
1.5
0.9
0.3
-0.3
4 6 8 10 12 14 16 18 20 22 24 26
PERIOD

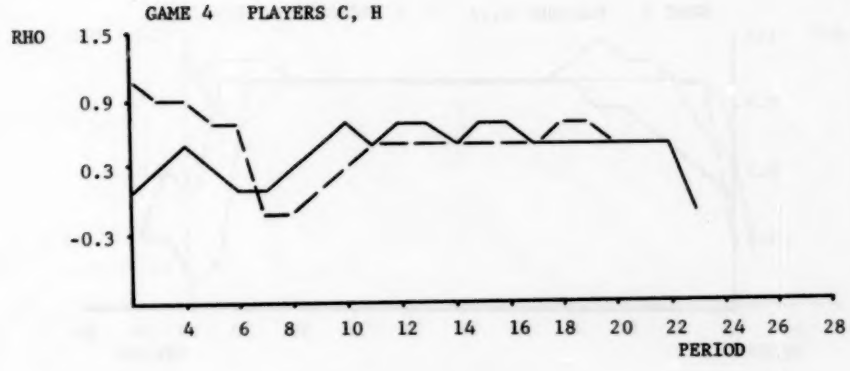

GAME 2 PLAYERS E, F, G, H
RHO
1.5
0.9
0.3
-0.3
4 6 8 10 12 14 16 18 20 22 24 26
PERIOD

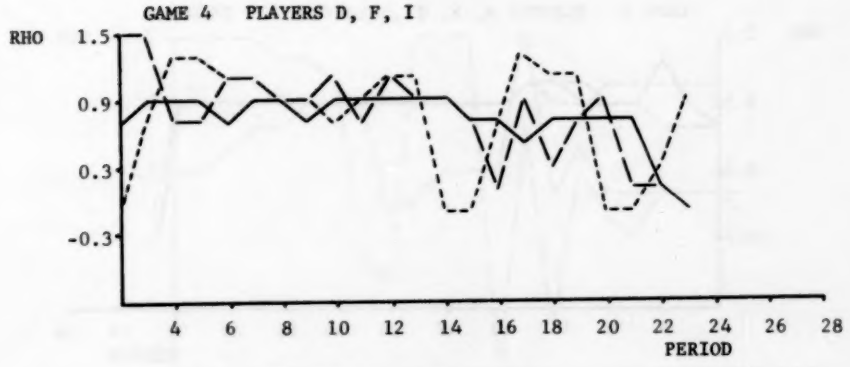

GAME 3 PLAYERS A, E
RHO
1.5
0.9
0.3
-0.3
-0.9
4 6 8 10 12 14 16 18 20 22 24 26 28
PERIOD

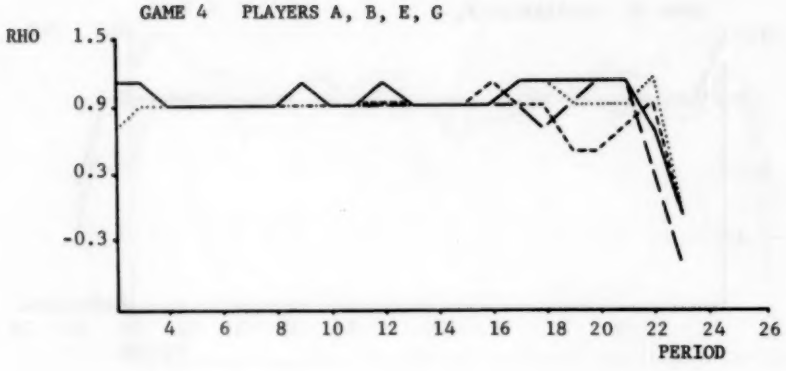

GAME 3 PLAYERS C, D
RHO
1.5
0.9
0.3
-0.3
4 6 8 10 12 14 16 18 20 22 24 26 28
PERIOD

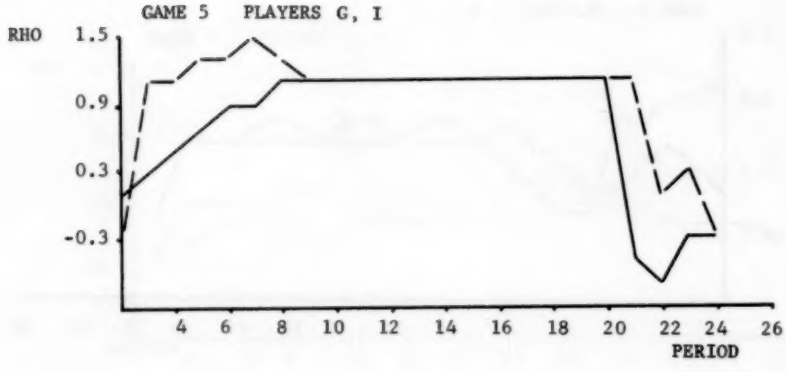

GAME 3 PLAYERS H, I
RHO
1.5
0.9
0.3
-0.3
4 6 8 10 12 14 16 18 20 22 24 26 28
PERIOD

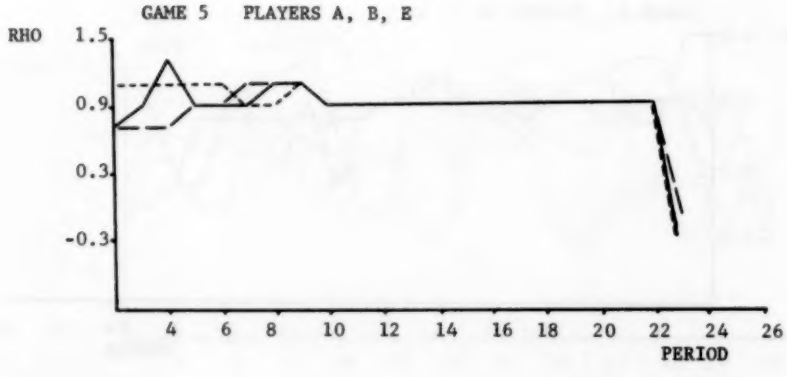

GAME 3 PLAYERS B, F, G
RHO
1.5
0.9
0.3
-0.3
4 6 8 10 12 14 16 18 20 22 24 26 28
PERIOD

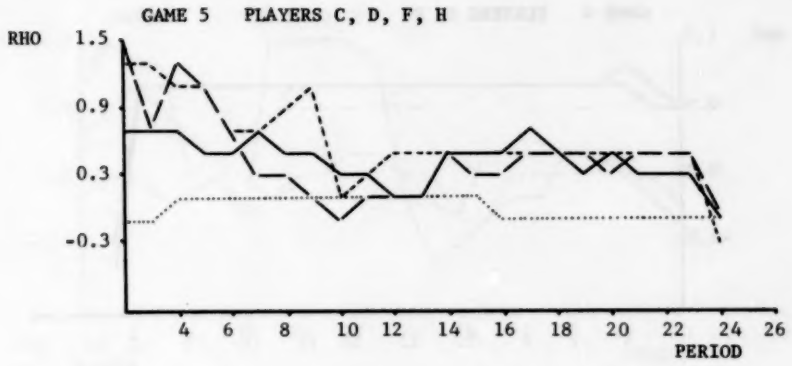

GAME 4 PLAYERS C, H
RHO
1.5
0.9
0.3
-0.3
4 6 8 10 12 14 16 18 20 22 24 26 28
PERIOD

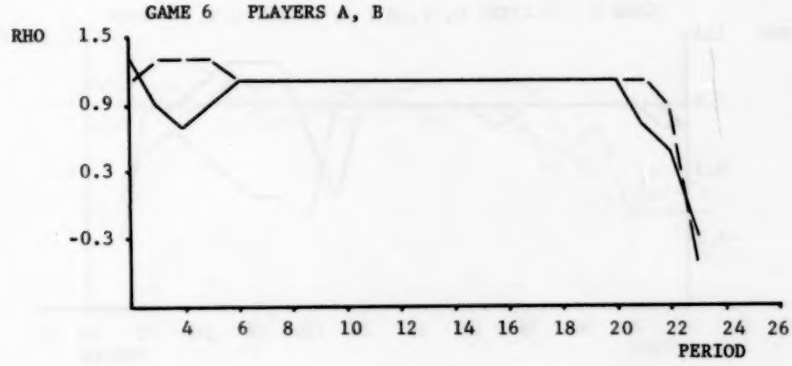

GAME 4 PLAYERS D, F, I
RHO
1.5
0.9
0.3
-0.3
4 6 8 10 12 14 16 18 20 22 24 26 28
PERIOD

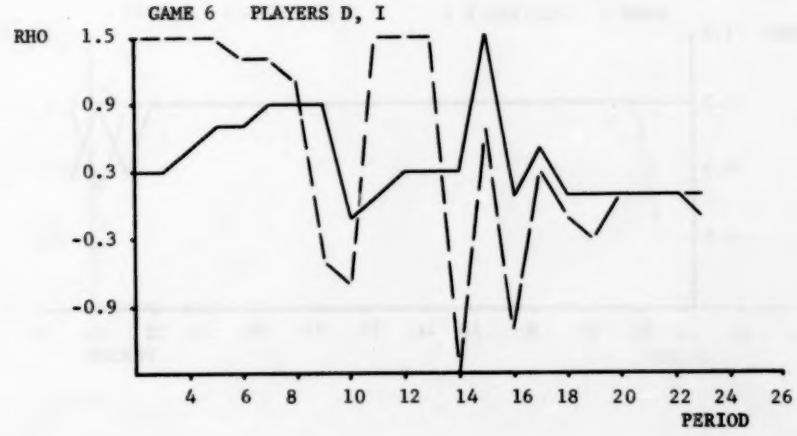

GAME 4 PLAYERS A, B, E, G
RHO
1.5
0.9
0.3
-0.3
4 6 8 10 12 14 16 18 20 22 24 26
PERIOD

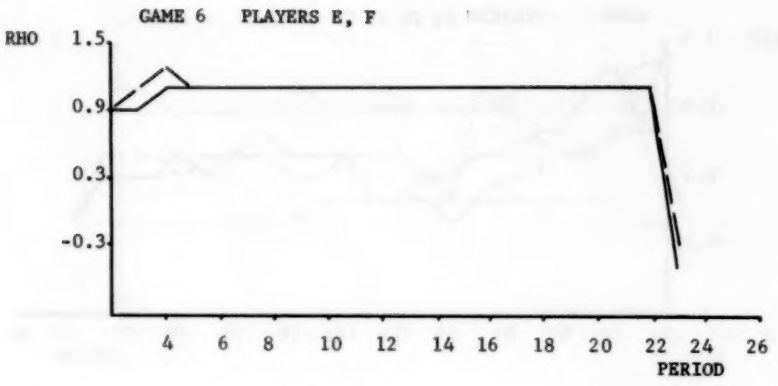

GAME 5 PLAYERS G, I
RHO
1.5
0.9
0.3
-0.3
4 6 8 10 12 14 16 18 20 22 24 26
PERIOD

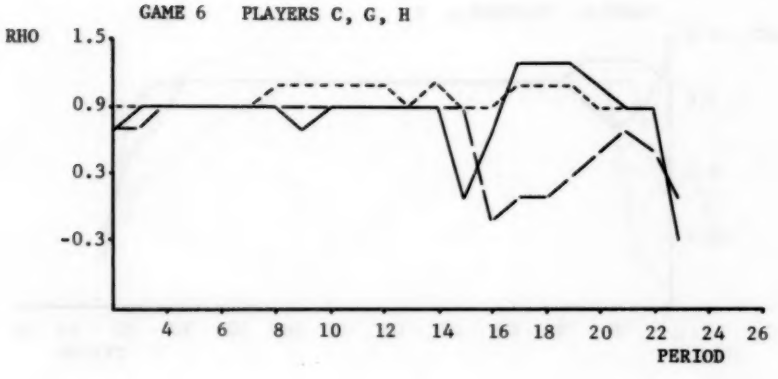

GAME 5 PLAYERS A, B, E
RHO
1.5
0.9
0.3
-0.3
4 6 8 10 12 14 16 18 20 22 24 26
PERIOD

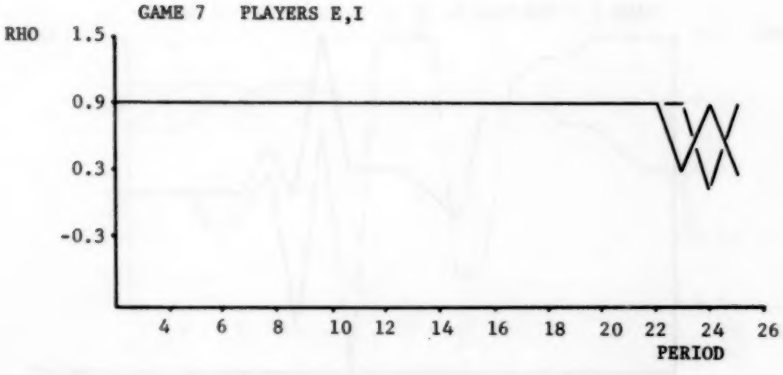

GAME 5 PLAYERS C, D, F, H
RHO
1.5
0.9
0.3
-0.3
4 6 8 10 12 14 16 18 20 22 24 26
PERIOD

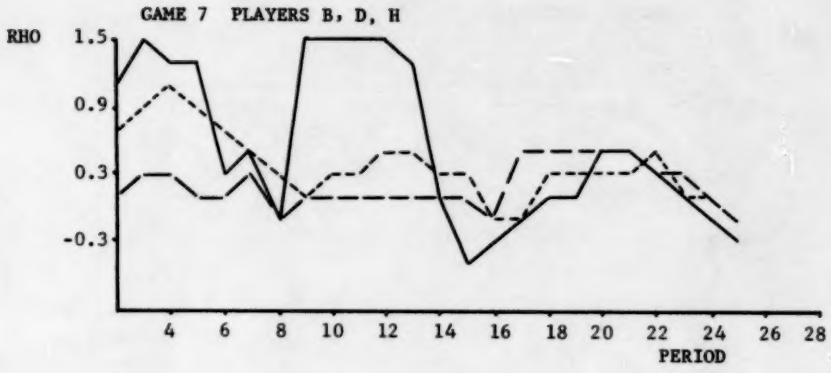

GAME 6 PLAYERS A, B
RHO
1.5
0.9
0.3
-0.3
4 6 8 10 12 14 16 18 20 22 24 26
PERIOD

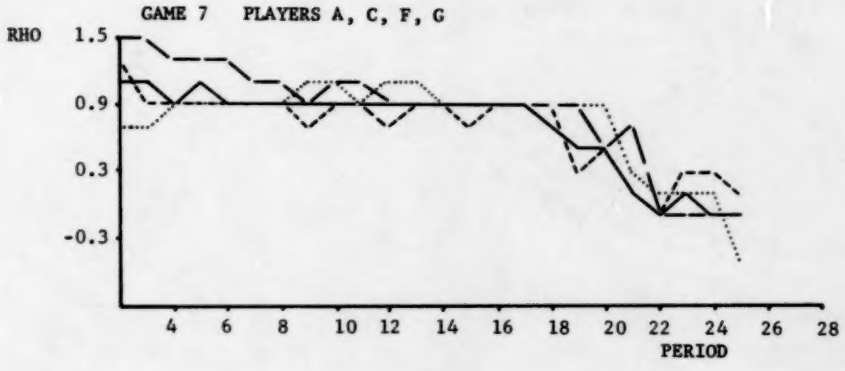

GAME 6 PLAYERS D, I
RHO
1.5
0.9
0.3
-0.3
-0.9
4 6 8 10 12 14 16 18 20 22 24 26
PERIOD

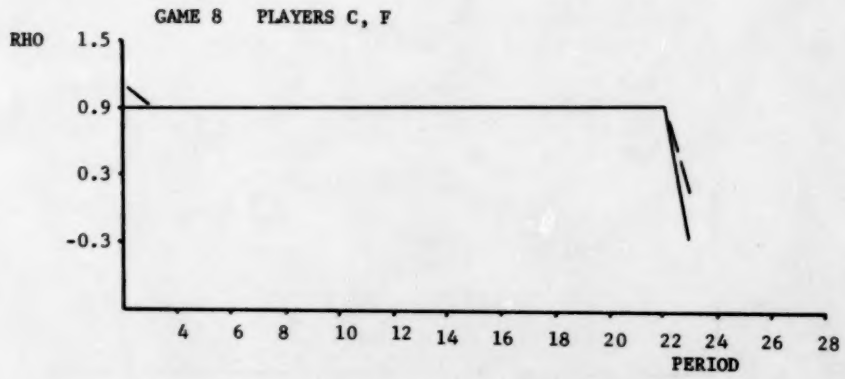

GAME 6 PLAYERS E, F
RHO
1.5
0.9
0.3
-0.3
4 6 8 10 12 14 16 18 20 22 24 26
PERIOD

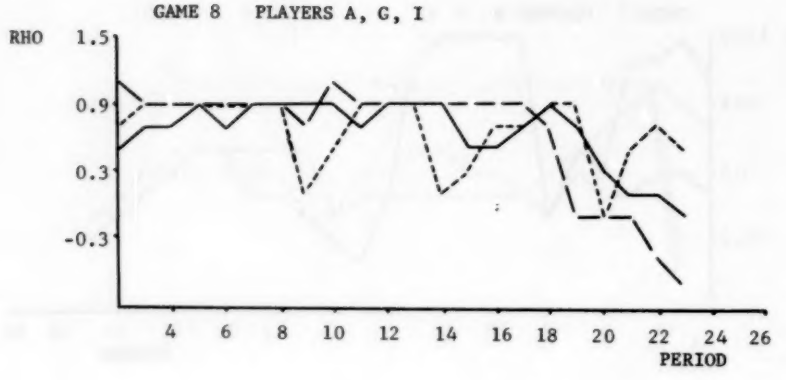

GAME 6 PLAYERS C, G, H
RHO
1.5
0.9
0.3
-0.3
4 6 8 10 12 14 16 18 20 22 24 26
PERIOD

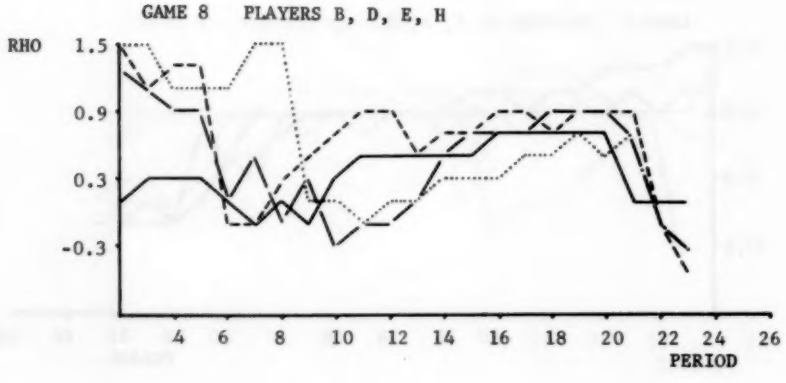

GAME 7 PLAYERS E,I
RHO
1.5
0.9
0.3
-0.3
4 6 8 10 12 14 16 18 20 22 24 26
PERIOD

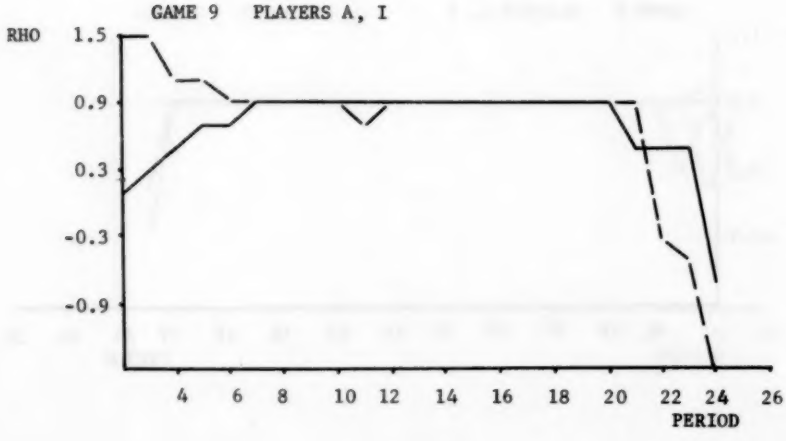

GAME 7 PLAYERS B, D, H
RHO
1.5
0.9
0.3
-0.3
4 6 8 10 12 14 16 18 20 22 24 26 28
PERIOD

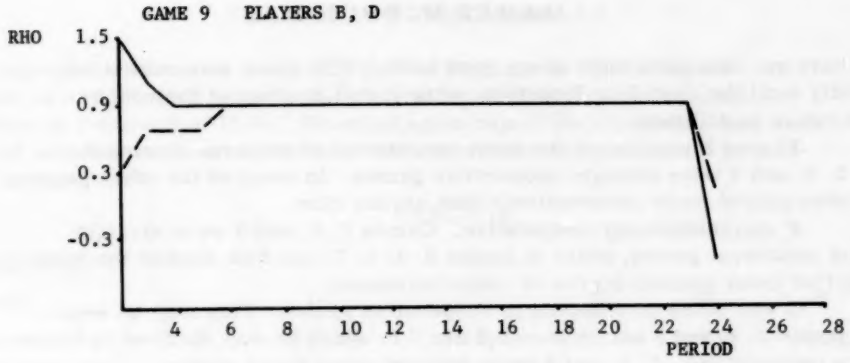

GAME 7 PLAYERS A, C, F, G
RHO
1.5
0.9
0.3
-0.3
4 6 8 10 12 14 16 18 20 22 24 26 28
PERIOD

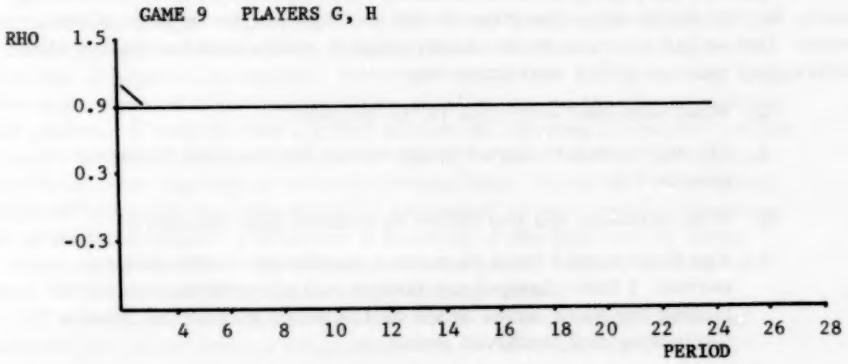

GAME 8 PLAYERS C, F
RHO
1.5
0.9
0.3
-0.3
4 6 8 10 12 14 16 18 20 22 24 26 28
PERIOD

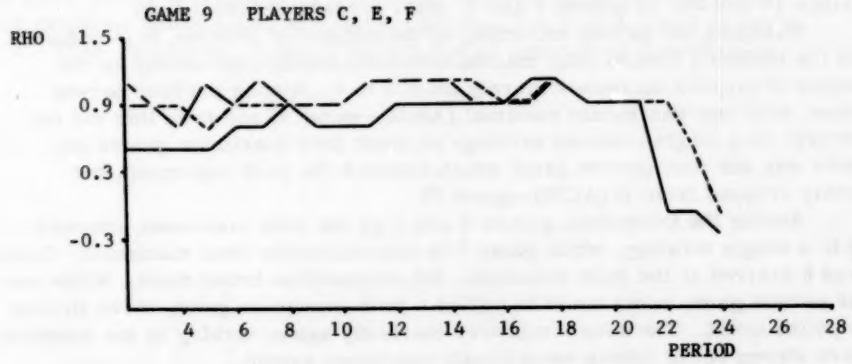

GAME 8 PLAYERS A, G, I
RHO
1.5
0.9
0.3
-0.3
4 6 8 10 12 14 16 18 20 22 24 26
PERIOD


GAME 8 PLAYERS B, D, E, H
RHO
1.5
0.9
0.3
-0.3
4 6 8 10 12 14 16 18 20 22 24 26
PERIOD


GAME 9 PLAYERS A, I
RHO
1.5
0.9
0.3
-0.3
-0.9
4 6 8 10 12 14 16 18 20 22 24 26
PERIOD


GAME 9 PLAYERS B, D
RHO
1.5
0.9
0.3
-0.3
4 6 8 10 12 14 16 18 20 22 24 26 28
PERIOD


GAME 9 PLAYERS G, H
RHO
1.5
0.9
0.3
-0.3
4 6 8 10 12 14 16 18 20 22 24 26 28
PERIOD


GAME 9 PLAYERS C, E, F
RHO
1.5
0.9
0.3
-0.3
4 6 8 10 12 14 16 18 20 22 24 26 28
PERIOD

to have you take advantage of my good faith." The game then oscillated wildly until the final 6 or 7 periods, when it settled down at the noncooperative equilibrium.

Player E was one of the most cooperative of players. Games 3, 4, 5, 6, and 7 were straight cooperative games. In many of the other games, E often played more cooperatively than anyone else.

F was moderately cooperative. Games 3, 6, and 8 were straight joint maximum games, while in games 2, 4, 5, 7, and 9 he showed the average (for those games) degree of cooperativeness.

G was prone to undercut in cooperative games. This may be seen in games 2, 4, and 8 and in games 6 and 7 in which he was the first to follow C in undercutting. 3, 5, and 9 were straight cooperative games.

H started playing one way and then changed his strategy. For the first two games, he played strictly noncooperatively, then, in the remaining games, he was about as cooperative as the average player in each of the games. One of his answers on the questionnaire shows that he realized his earlier play was not in his best interests.

Q: What was your objective in the project?

A: My object was to earn enough money for current living expenses.

Q: What strategy did you follow to achieve this objective?

A: The first night I tried to make a maximum profit on each period. I then changed my tactics and concentrated on maintaining the same price scale as the other players to avoid a fluctuating and cutthroat game.

Player I showed a liking for going to the joint maximum, and then undercutting. For example, note game 3, periods 13, 18, and 23; game 4, periods 14 and 20. In games 7 and 9, other players undercut first.

Grouping the games according to the number of players, it is apparent that the tendency toward joint maximization diminishes noticeably as the number of players increases merely from 2 to 4. Among the four person games, only one was jointly maximal (ABEG—game 4) and even this did not converge to a single constant strategy as most joint maximum games did. There was one four person game which attained the joint maximum, but quickly dropped from it (ACFG—game 7).

Among the triopolies, games 3 and 5 go the joint maximum, converging to a single strategy, while game 9 is approximately joint maximum. Games 4 and 6 arrived at the joint maximum, but cooperation broke down. While one four person game in six could be called a joint maximum game, three in nine triopolies could. The score improves markedly again, turning to the duopolies where eleven out of fifteen were jointly maximum games.

The data for the end effect analysis were computed in the same manner for the second series of games as for the first. Table 18 shows the actual

and expected frequencies of the last period rho values. The computed chi-square is 337.7, which exceeds the critical value (for the 99% level and 6 degrees of freedom) of 18.48. We must again reject the hypothesis that final period behavior does not differ from the behavior of the period just preceding the last one.

TABLE 18

Frequencies							
Actual	57	14	2	2	0	6	0
Expected	(8.2)	(11.8)	(6.2)	(12.0)	(6.6)	(26.6)	(9.6)

In summary, it is apparent that these experiments enforce certain of Fouraker and Siegel's findings. We also found great differences in the outcome of complete information duopoly games and fewer differences as the number of players increased. However, we did not find as strong a general tendency toward the noncooperative equilibrium as that found in three person games. It may be that a great amount of learning takes place in the first game in which a subject plays; so much, in fact, that the first game and the later ones may not be strictly comparable. Much of the variation in subjects' behavior was successfully explained by our model of behavior which made each subject's behavior a function of the behavior of those with whom he was paired, the structure of the game, and the number of players. It seems all these factors may be important.

Of course, there remains much work to be done in experimental economics just in the area of oligopoly. The level of contact between players might be varied, side payments could be allowed, the number of players could be allowed to get quite large—say 10, parameter changes in the game model could be more extensive, subjects from various populations might be tested under similar conditions, and games could be asymmetric. This, of course, is only a partial list of interesting possibilities.

Summary: Von Neumann and Morgenstern have formulated a set of plausible axioms which imply that an individual, when confronted with choice situations involving risk, should maximize expected utility. However, the behavior of many individuals has been found to be inconsistent with the Neumann-Morgenstern axioms, strictly interpreted. In this study, a model of choice is formulated which permits random deviations from expected utility maximization. The probability of such a deviation in a choice between two "lotteries" is hypothesized to be a function of the difference in expected utility between those two lotteries.

The predictive power of the model was tested in an experiment involving ten subjects. Each subject was asked to make choices in pairs of two-payoff lotteries. The lotteries were constructed from ten monetary payoffs ranging in value from $9.75 to -$1.50. Choices were used for two purposes—estimation and prediction. The choices observed in estimation lottery pairs were used to obtain maximum likelihood estimates of the utilities of the ten payoffs. The choices observed in the prediction lottery pairs were used to test the predictive ability of expected utility maximization against two alternative prediction techniques—maximization of expected value (actuarial choice) and maximization of the minimum payoff (minimax choice). The experimental results indicate that the expected utility hypothesis leads to predictions which are statistically superior to actuarial maximization for 7 of the 10 subjects and to minimax choice for 8 of the 10 subjects.

This essay was submitted to Yale University in 1963 in partial fulfillment of the requirements for the degree of Doctor of Philosophy. The author is currently Assistant Professor of Economics at Carnegie Institute of Technology.

Dissertation Committee: Professors William J. Fellner (Chairman), James Tobin, and Edmund S. Phelps.

*

During the preparation of this study, I have received assistance—both substantive and editorial—from many people. In particular, I wish to thank my committee—Professors William J. Fellner, James Tobin, and Edmund S. Phelps. I am especially indebted to William C. Brainard who spent many hours discussing conceptual and expositional difficulties. At varying stages of the study, I have benefited from comments by Donald W. Taylor (of the Department of Psychology at Yale), John F. Muth, James W. Friedman, T. N. Srinivasan, and Alan S. Manne.

The experiment was financed by the Cowles Foundation for Research in Economics at Yale. My wife, Ann, assisted with tabulation of experimental results. Bernard P. Dzielinski of IBM provided computational facilities and programming assistance. Additional computer time was provided by the Computation Center at the Massachusetts Institute of Technology and by the Cowles Foundation.

F. T. D.

INDIVIDUAL CHOICE UNDER UNCERTAINTY—AN EXPERIMENTAL STUDY

F. Trenery Dolbear, Jr.

1. Estimation Techniques

1.1 "Neumann-Morgenstern"

If an individual behaved strictly and consistently in conformity with the Neumann-Morgenstern axioms,[1] estimation of his utility function could be obtained by the following procedure. He could be asked to state the payoff, x dollars ($x), which would make him indifferent between participation and non-participation in a lottery which offers $x with probability 1/2 and minus one dollar (−$1) with probability 1/2. He would select x so that the expected utility of participation is equal to the expected utility of non-participation:[2]

$$\frac{1}{2}\,U(\$x) + \frac{1}{2}\,U(-\$1) = 1\,U(\$0)$$

Since Neumann-Morgenstern utility is unique only up to a linear transformation, two utilities may be assigned arbitrarily.[3] If the utilities 0 and −1 are assigned to payoffs $0 and −$1 respectively, then the utility of $x = + 1. Utility estimates for additional payoffs could be obtained by presenting the indi-

---

1. See John von Neumann and Oskar Morgenstern, Theory of Games and Economic Behavior (3d ed.; Princeton: Princeton University Press, 1953), pp. 26–27.

2. The risk-neutral or actuarial individual would indicate x = $1 while the risk-averter (risk-lover) would indicate an amount above (below) $1.

3. The expected utility hypothesis suggests that if an individual ranks three objects in order C, A, B, and is indifferent between receiving A with certainty or a lottery in which he will receive either C with probability α or B with probability $1 - \alpha$, then α is an estimate of the ratio of the preference of A over B to that of C over B (see Neumann and Morgenstern, op. cit., p. 18). For convenience, we may write (U ( ) designates the utility function):

$$U(A) = \alpha U(C) + (1 - \alpha)\,U(B) \text{ which is equivalent to } \frac{U(A) - U(B)}{U(C) - U(B)} = \alpha$$

The utility assignments U(x) can be transformed into assignments V(x) = a + b U(x), b > 0, without affecting the analysis. In the example above:

$$\frac{V(A) - V(B)}{V(C) - V(B)} = \frac{(a + b\,U(A)) - (a + b\,U(B))}{(a + b\,U(C)) - (a + b\,U(B))} = \frac{(U(A) - U(B))}{(U(C) - U(B))}$$

vidual with similar choices. For example, the payoff $y could be requested such that:

$$\frac{1}{2}\,U(\$y) + \frac{1}{2}\,U(\$0) = 1\,U(\$x)$$

The utility of $y is equal to +2. A trivial variant of this estimation procedure would use probability as the choice variable (with the payoffs completely specified).

While theoretically the above techniques are both simple and efficient (i.e., a large number of estimates can be obtained with a small number of responses—compared with the estimation techniques discussed below), in practice they are of little value. The reason for this fact is that the choices of most individuals are not consistent with the Neumann-Morgenstern axioms. If the axioms are met, the utility estimates obtained by the above procedures would be independent of the order in which they are obtained. However, for many individuals, juggling the sequence of choices would lead to different utility estimates, and these techniques could not choose among them.

In general, data for estimation should be obtained from choice situations which resemble in structure the situations for which predictions are desired. Usually, we wish to predict choices among alternative lotteries, each of which is completely specified by payoffs and associated probabilities; however, in the "Neumann-Morgenstern" procedure, discussed above, the individual is required to complete a lottery. Both situations involve the individual in introspection, but the introspective process required by the "Neumann-Morganstern" procedure is more difficult. For example, an individual may have little difficulty in choosing one bag of potato chips over 10 sticks of chewing gum, but he may find himself at a loss when asked how many more sticks of chewing gum would be required to make him indifferent between the two prospects. When confronted with the latter question, the individual may for convenience (to the experimenter's dismay) adopt some simple, arbitrary rule divorced from the utility maximization calculations the individual is presumed to make in real choice situations.

The remainder of this section will briefly describe the experimental approaches of various authors to utility estimation. All of these approaches meet the objections to the naive "Neumann-Morgenstern" estimation procedure. They confront the individual with completely specified alternatives, and they can accommodate "inconsistent" choices.

1.2 Mosteller and Nogee

Mosteller and Nogee, who estimated utilities for fourteen subjects, were the first to attempt to obtain a utility assignment from an individual's choices between alternatives.[1] The following is a description of their technique.

1. Frederick Mosteller and Philip Nogee, "An Experimental Measurement of Utility," Journal of Political Economy, Vol. 59 (1951), pp. 371–404.

A subject was offered a series of lotteries, each of which contained a gain of $x with probability p and a loss of $.05 with probability 1−p. The subject could accept the lottery (and subsequently receive $x or pay $.05), or he could reject the lottery at no cost. The subjects were confronted with various values of x for a given p. Thus for p = 1/6, the subject was asked to accept lotteries such as:

1/6	.19	,	1/6	.28	, and	1/6	.43	, where	1/6	.19
5/6	−.05		5/6	−.05		5/6	−.05		5/6	−.05

means a 1/6 chance of winning $.19 and a 5/6 chance of losing $.05, etc. The subject was confronted with each x,p pair a number of times over the course of the experiment.

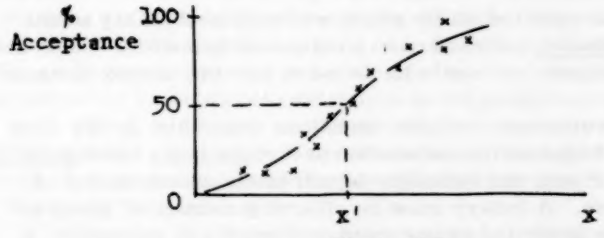


Figure 1.1

Figure 1.1 shows the relative frequency of acceptance of lotteries, with a given p, plotted as a function of x. As expected, the plotted points suggested that frequency of acceptance was a monotonic increasing function of the payoff x. A "freehand" or "broken-line" curve was then fitted to these points. The value of x (in Figure 1.1 designated as x') which would have presumably led to fifty percent acceptance was found. It was assumed that equal frequency of acceptance and rejection of a lottery implies indifference between participation and non-participation. That is, the expected utility of participation in the lottery with x' would be equal to the expected utility of non-participation:

$$p\ U(\$x') + (1-p)\ U(-\$.05) = 1\ U(\$0)$$

The payoffs −$.05 and $0 were arbitrarily assigned utilities −1 and 0, respectively; consequently the utility of x' must be (1−p)/p. This estimation procedure was repeated once for each of seven values of p.

In contrast to the estimation procedures described in the first section, the Mosteller-Nogee estimation procedure thrives on inconsistent choices. Their construction suggests a stochastic theory of choice in which probabilities depend upon differences in expected utility between the alternatives. If

an individual's behavior was not stochastic, i.e., if he always selected the alternative with the higher expected utility, Figure 1.1 would be replaced by Figure 1.2. Unless the experimenter had been fortunate enough to select

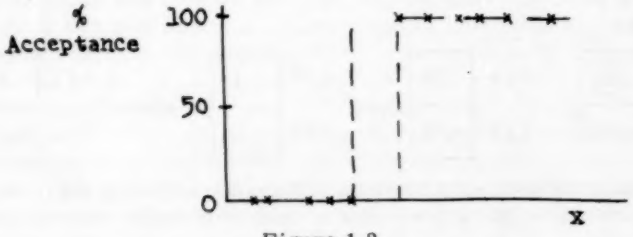


Figure 1.2.

two alternatives of equal expected utility which with high probability would result in a frequency between zero and one, a unique estimate of x' could not be obtained by the technique. x' would lie between the two dotted lines in Figure 1.2.

Although an improvement over the technique described in the first section, the Mosteller-Nogee utility estimation procedure has a number of undesirable features.[1] First, the technique is extremely uneconomical of data, and data are scarce. A lottery must be offered a number of times to obtain a frequency which is plotted as one point in Figure 1.1; moreover, a number of such points are required before a curve can be fitted. The end result of this curve is an estimate of one point on the utility function. This procedure must be repeated for each utility point.

Second, Mosteller and Nogee always offered lotteries involving a −$.05 payoff against the non-participation payoff, $0. Choice data which lack one or both of these payoffs were not incorporated into the estimation procedure. The Mosteller-Nogee estimation technique does not require the use of −$.05 and $0; in particular, a payoff obtained as one of the first estimates could be used in obtaining later estimates. However, this restriction avoided or concealed a difficulty which casts doubt on the whole technique.

Suppose $.10 had been estimated to have a utility of +1 by the standard procedure (U(−$.05) = −1 and U($0) = 0). Further, suppose data were available so that each of the following choice indifferences could be estimated.

---

1. Donald Davidson, Patrick Suppes, and Sidney Siegel, Decision Making: An Experimental Approach (Stanford: Stanford University Press, 1957), pp. 21–23, contains extended discussion of some difficulties in the Mosteller-Nogee formulation.

1.) $1/4\ U(\$x') + 3/4\ U(-\$.05) = 1\ U(\$0)$

2.) $1/2\ U(\$x') + 1/2\ U(-\$.05) = 1\ U(\$.10)$

3.) $1/3\ U(\$x') + 2/3\ U(\$0) = 1\ U(\$.10)$

The reader can easily verify that the utility associated with $x' is +3 in each of the three situations; consequently estimates of $x' should be identical in 1.), 2.) and 3.). However, neither the estimation procedure nor actual behavior guarantees this equality; and if it does not occur, the procedure provides no way to resolve the ambiguity.

There is a related difficulty—estimates might be obtained which fail to indicate a monotonic utility function; for example, $.25 might be assigned a utility of +5 while $.30 was assigned a utility of +4. Again the estimation procedure is of no assistance. Such problems are unlikely to arise in the standard procedure involving −$.05 and $0. Monotonicity is insured if increasing the probability of receiving −$.05 leads to an increase of x'. Most subjects will display this degree of consistency. Thus while the Mosteller-Nogee technique appears to handle inconsistent choices, in fact this ability is achieved by an artificial restriction in the presentation of choice alternatives.

Third, Mosteller and Nogee require a subject to choose between a gamble and a certainty. Whereas the model assumes that choices are based only on expected utility of the payoffs, choices also reflect the "utility" or "disutility" of gambling.[1]

Finally, the Mosteller-Nogee technique requires a wide variety of probabilities and assumes that a subject's subjective probabilities are equal to the objective probabilities. Experimental evidence suggests that most individuals do not view probabilities objectively.[2]

1.3 Davidson and Suppes

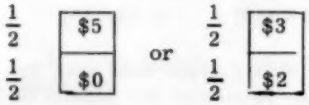
In contrast to the Mosteller-Nogee procedure, Davidson-Suppes' method of utility estimation (i.e., their linear programming method) is not restricted to a special data format. They requested their subjects to choose between two, two-payoff lotteries. Although the objects of choice were long-playing records, their estimation procedure in no way depends upon this feature of the experiment. Therefore, it can be discussed as if the payoffs were monetary.

---

1. Davidson-Suppes-Siegel and the present experimenter confront subjects with two gambles so that the act of gambling cannot be avoided. Of course, even in these circumstances the two lotteries (gambles) will not be equal with respect to risk.

2. See Ward Edwards, Articles on probability preferences, American Journal of Psychology, Vol. 66 (1953), pp. 345–64; Vol. 67 (1954), pp. 56–67 and 68–95.

For example, a subject might be presented with a choice between two lotteries,



If the subject selects the left lottery, the experimenter can infer from this choice that the expected utility of the left lottery is no less than the expected utility of the right lottery, i.e.

$$\frac{1}{2}\ U(\$5) + \frac{1}{2}\ U(\$0) \geq \frac{1}{2}\ U(\$3) + \frac{1}{2}\ U(\$2)$$

Each choice of this type provides information on the subject's utility function. For the example shown here, we know (for a Neumann-Morgenstern man) that the utility assigned to $5 plus the utility assigned to $0 must not be less than the utility assigned to $3 plus the utility assigned to $2. (In this example probabilities cancel.) Similar restrictions on the utility function can be obtained from other choices.

If a subject violates the Neumann-Morgenstern axioms, there will be no assignment of utilities to payoffs which is consistent with these restrictions. Therefore, a criterion must be provided which will choose among utility assignments none of which is strictly consistent with expected utility maximization.

Suppose a utility assignment for the various payoffs is arbitrarily selected, and the algebraic expected utility difference (EUD) is computed between the chosen lottery and the rejected lottery for each choice of the subject. For example, a utility assignment might suggest:

Choice Number	(E. U. Chosen)	–	(E. U. Rejected)	=	EUD
1.	22	–	4	=	18
2.	3 1/2	–	1 1/2	=	2
3.	9	–	9	=	0
4.	15	–	6	=	9
5.	4 3/4	–	4 1/4	=	1/2
6.	4	–	10	=	−6
7.	−1	–	−8 2/3	=	7 2/3
8.	20	–	29	=	−9

Negative numbers in the EUD column indicate apparent inconsistencies. A similar table could be calculated for any utility assignment. If an assignment can be found which results in a series of expected utility differences which contains no negative numbers, the subject has displayed behavior consistent

with the Neumann-Morgenstern axioms. However, in general no such utility assignment can be found; therefore, Davidson-Suppes select a criterion which will choose among utility assignments, none of which can "explain" all choices as maximization of expected utility.

Presumably, utility assignments which suggest "less" inconsistency should be preferred to assignments which suggest "more" inconsistency. Just what is meant by "less" and "more," however, is a difficult question, as will be seen below. Davidson and Suppes select that utility assignment which minimizes the largest apparent inconsistent choice as measured by expected utility difference. Consider a utility assignment which implies the following EUD's: −3, 16, −2 1/2, −8, 12, 1 1/4, 5, 7. The minimum EUD is −8. Davidson-Suppes' criterion would prefer the utility assignment associated with this series to the utility assignment which gave rise to the series above which has a minimum EUD of −9.

Davidson and Suppes rationalize their criterion by reference to a "threshold of preference" (in terms of expected utility difference) which they call Θ. If two lotteries differ in expected utility "by more than Θ, the larger [in E.U.] must be chosen. If the difference is within Θ the choice is stochastic."[1] The maximum minimum EUD is the Davidson-Suppes estimate of Θ, Thus, any apparent inconsistent choice will appear to have an expected utility difference less than or equal to Θ.

The logical connection between ascribing a "threshold of preference" to an individual and an estimation procedure based on minimization of this threshold is elusive. The criterion is an experimental construct which was used to find utility estimates for predictive purposes. It has little basis in a theoretical model of choice.

Estimation of utilities using the criterion outlined above requires a solution of a linear programming problem. Davidson and Suppes add the parameter Θ to the expected utility of each chosen lottery. For example, the illustration above would be amended to read:

$$\frac{1}{2}\,U(\$5) + \frac{1}{2}\,U(\$0) + \Theta \geq \frac{1}{2}\,U(\$3) + \frac{1}{2}\,U(\$2)$$

That is, the expected utility of the chosen lottery plus Θ must not be less than the expected utility of the rejected lottery. Thus Θ must take on a value large enough so that none of the inequalities is violated.[2]

---

1. Davidson, Suppes, and Siegel, op. cit., p. 85.

2. Davidson-Suppes actually canceled the probabilities and wrote their inequalities in "atomic" utility intervals, e.g. [U($5)−U($3)], [U($3)−U($2)]. The estimation results would be identical for either approach.

From such inequalities, a linear programming problem is formulated as follows. The result of confronting the subject with M choices between two lotteries will be M linear restraints. The utility of each payoff (e.g., $5, $3, $2, $0) constitutes an activity, and the level of operation of that activity is the utility estimate. The coefficient matrix for the restraints is obtained from the probabilities. Since each payoff will appear in several lottery pairs, each activity (utility) will have a non-zero coefficient in several restraints. Finally, the objective function, containing a coefficient of -1 for Θ, is maximized when Θ is minimized.

If all payoffs are assigned the same utility, then expected utility differences become zero, and, consequently, such a solution is "optimal." However, Davidson-Suppes restrained all utility intervals to be no less than +1 (e.g., $U(\$5) - U(\$3) \geq +1$), which insures that the utility function will be strictly increasing. Since expected utility differences do not depend upon the "origin" of the utility function, a utility can be assigned arbitrarily to one of the payoffs.

Davidson and Suppes were concerned that the optimal utility estimates obtained with their criterion are not, in general, unique:[1]

> The minimum Θ is compatible with a convex polyhedron of solutions. The best single choice for [the utility estimates] is probably the centroid of this convex polyhedron. Due to limited computational facilities, we actually used the first solution obtained.

Davidson and Suppes seem to suggest that their criterion is not complete, i.e., some "optimal" solutions are more optimal than others. The following observation lends support to the view that the objective function was incorrectly specified. The Davidson-Suppes criterion requires minimization of the largest "error." If there are n utilities to estimate, it can be shown that the computation procedure implies that this largest error occurs in $n+1$ of the experimental choices.[2]

---

1. Davidson, Suppes, and Siegel, op. cit., p. 85.

2. Optimal solutions generated by linear programming algorithms are basic solutions (i.e., the number of activities in the solution equals the number of restraints). Thus, for M lottery pairs (restraints) and n utilities to estimate (activities), the optimal solution should typically contain the n utility estimates, Θ, and $M - (n+1)$ slack activities. Therefore, in $n+1$ restraints, the slack activity will be zero. This means that the optimal linear programming solution will assign utilities so that at least $n+1$ choices appear inconsistent by exactly Θ.

1.4 Manne

The Davidson-Suppes threshold criterion offends intuition because it is not concerned with the number of inconsistent choices. A utility assignment which implies a large number of inconsistent choices will be preferred to an assignment with very few inconsistent choices if the former has a lower Θ than the latter. For example, consider two utility assignments U_1 and U_2 which imply the following differences in expected utility:

U_1: 10, 0, −8, 26, 13, 3, 9

U_2: 0, 3, −2, 0, −7 3/4, 0, −4

The Davidson-Suppes criterion would of course choose U_2, but most observers would probably prefer U_1.

Examples such as these led Professor Alan Manne to suggest minimization of the sum of the negative numbers in the series of EUD's. This formulation also leads to a linear programming problem. A typical restraint takes the form:

$$\frac{1}{2}\,U(\$5) + \frac{1}{2}\,U(\$0) + \Theta_i \geq \frac{1}{2}\,U(\$3) + \frac{1}{2}\,U(\$2)$$

The structure follows the Davidson-Suppes formulation except there is now a separate Θ for each choice. Finding the optimal solution involves minimization of Θ_i, i.e. the sum over inconsistent choices of differences in expected utility between the two lotteries. Of course the Θ_i are restrained to be non-negative. (Manne also suggested minimization of $\Sigma\Theta_i^2$ which requires solution of a quadratic programming problem.)

The Manne linear criterion and the Davidson-Suppes criterion were compared in an unpublished experiment with six subjects, conducted at Yale during the summer of 1960 by Gordon Becker, William Brainard, Alan Manne, and this writer. The experimental technique was very similar to that used by Davidson-Suppes, except that small amounts of money were used as payoffs (ranging from \$.98 to −\$.49).[1]

Prediction based upon "Davidson-Suppes" and "Manne" utility estimates were not significantly different from each other, nor, indeed, were they different from a naive actuarial model of choice. The experiment was, however, instructive as to the inadequacies of both estimation criteria. These criteria both concern only the minimization of apparent inconsistency, and such mini-

---

1. This group did not follow Davidson-Suppes in requiring that the utility function be strictly increasing (see above); rather it arbitrarily assigned utilities to two payoffs and required that the utility function be non-decreasing. In section 1.5 the two "normalization" techniques are discussed in more detail.

mization frequently results in a utility assignment implying that the subject is indifferent in a large proportion of choices. This result seems contrary to a priori notions about preferences. When a subject is confronted with a variety of choices, it would be surprising if many did not involve large differences in expected utility. Similarly, it would be surprising if the subject were exactly indifferent in many of the choices. This suggests the desirability of an estimation criterion which—whatever its other characteristics—makes obvious choices look obvious.

Although there are a number of ad hoc criteria which might be used in choosing among utility assignments, the difficulties discussed above reflect the consequences of arbitrarily selecting any one criterion. Professor Jacob Marschak suggested that a stochastic model of choice would provide its own estimation criterion. Such a model, and an estimation procedure based upon it, are presented in the next section.

Below is a discussion of a further difficulty in the Davidson-Suppes and Manne estimation techniques. Since the difficulty has been eliminated in the present study, it is possible for the reader to skip to the next section of the essay without loss of continuity.

1.5 A Problem in the Definition of Inconsistency

Both the Davidson-Suppes and the Manne criteria involve minimization (in some sense) of certain expected utility differences. However, since utility functions are unique only up to a linear transformation, any utility function can be legitimately transformed so as to make expected utility differences approach zero. In the limit, all payoffs are assigned the same utility. Comparisons of utility functions belonging to different families (i.e., not related by linear transformation) are then meaningless, since any "family" can appear "optimal" if an appropriate member of the family is chosen to calculate expected utility differences.

To avoid this indeterminacy, Davidson-Suppes restrained utility intervals between any two payoffs to be no less than +1. This restraint insures that the utility function will be strictly increasing. Thus, each "family" of utility functions will be represented in the optimality "contest" by the member which has one or more utility intervals equal to one and all the rest greater than one. This normalization may be seen to favor families of utility functions which are represented by a function with a large number of utility intervals equal to one. The results of Davidson and Suppes' Session I show approximately 80% of the interval estimates—between two adjacent payoffs—equal +1.[1]

1. Davidson, Suppes, and Siegel, op. cit., p. 95. To be sure, Davidson-Suppes seem aware of this difficulty (although they offer no solution) when they remark: "Without doubt this equality of intervals is to a considerable extent an artifact of the model. Subject to the normalizing assumptions made, the linear programming model imposes the simplest possible numerical structure on the data." (p. 95.)

Normalization in the Manne experiment took a different form. Utility assignments to two payoffs were made arbitrarily; consequently, each family of utility functions was "represented" by the member which passes through both of these utility assignments. While this normalization seems (to me) intuitively more appealing, again no justification can be provided.[1]

Perhaps the difficulty can be avoided by the following approach. Although for a given "family" of utility functions expected utility of the chosen lottery minus expected utility of the rejected lottery is not unique, the ratio of these expected utilities is unique. All members of a family of utility functions will give rise to identical ratios EU_c/EU_r. If $EU_c/EU_r \geq 1$, the choice is consistent; but if $EU_c/EU_r < 1$, the choice is inconsistent. A "threshold" parameter Θ could be introduced such that $\Theta\ (EU_c/EU_r \geq 1$ for all choices. Optimization would require finding the utility assignments which minimize Θ, but "comparison" of utility families would not be

---

1. A simple example will illustrate that the "optimal" utility function may not be invariant with this "normalization." Suppose an individual makes choices between lotteries which lead to the following inequalities—using the $\Sigma\Theta_i$ criterion:

$$(1)\ \frac{3}{4}\,U(\$100) + \frac{1}{4}\,U(\$0) \leq U(\$50) + \Theta_1$$

$$(2)\ \frac{1}{4}\,U(\$100) + \frac{3}{4}\,U(\$0) + \Theta_2 \geq U(\$50)$$

$$(3)\ \frac{1}{2}\,U(\$100) + \frac{1}{2}\,U(\$0) + \Theta_3 \geq U(\$50)$$

If he makes the first choice four times, the second choice twice, and the third choice three times, we wish to find utility assignments which minimize:

$$(4)\ 4\Theta_1 + 2\Theta_2 + 3\Theta_3$$

Let U($100) = 100 and U($0) = 0. Minimization of (4) gives U($50) = 50. Alternatively, let U($50) = 50 and U($0) = 0. Minimization of (4) gives U($100) = 66 2/3. Thus, the "optimal" Neumann-Morgenstern utility family is not independent of arbitrary "normalization."

based upon any arbitrary restriction.[1] A similar construction could be provided for the Manne criteria.

2. Theory and Estimation

2.1 A Model of Choice

A stochastic model of choice requires assumptions concerning sources of random variation in the choices of an individual. For choice between two lotteries, this stochastic influence may arise from characteristics of the lotteries—difference in expected utility or difference in dispersion of the payoffs.[2] Or the stochastic influence may arise from circumstances unrelated to the pair of lotteries. The individual may at random "look out the window" and select a lottery without regard to payoffs, or he may (again at random) make a reflex error and select the less desired lottery. For most individuals, "errors" (i.e., failure to choose so as to maximize expected utility) are probably caused by a combination of these factors.

Several authors have formulated axiomatizations which imply random variation in choices.[3] Typically, the strict transitivity axiom of Neumann-Mortenstern is replaced by an axiom specifying stochastic transitivity. That

---

1. This is a non-linear problem and is very similar to a technique suggested—for quite different reasons—by Patrick Suppes and Karol Walsh, "A Non-Linear Model for the Experimental Measurement of Utility," Behavioral Science, Vol. 4 (1959), pp. 204–11.

2. The model is not intended to apply to choices which are dominated or trivial. For example, consider a choice between two lotteries

$\frac{1}{2}$	\$5	and	$\frac{1}{2}$	\$3
$\frac{1}{2}$	\$2		$\frac{1}{2}$	\$0

This choice is "dominated" since $U(\$5) > U(\$3) > U(\$2) > U(\$0)$ implies

$$\frac{1}{2}U(\$5) + \frac{1}{2}U(\$2) > \frac{1}{2}U(\$3) + \frac{1}{2}U(\$0)$$

3. For example, Donald Davidson and Jacob Marschak, "Experimental Tests of a Stochastic Decision Theory," in Measurement: Definitions and Theories, C. West Churchman and Philburn Ratoosh, eds. (New York: John Wiley & Sons, Inc., 1959); G. Debreu, "Stochastic Choice and Cardinal Utility," Econometrica, Vol. 26 (1958), pp. 440–44; R. Duncan Luce, Individual Choice Behavior (New York: John Wiley & Sons, Inc., 1959); and Patrick Suppes, "Behavioristic Foundations of Utility," Econometrica, Vol. 29 (1961), pp. 186–202.

is, an individual is permitted to make "errors," either of evaluation or perception, but the probability of such errors is related to his preferences in a specific way. These stochastic models of choice are defined,[1] so that a utility function unique up to an increasing linear transformation implies that $P(A,B)$[2] $\leq P(C,D)$ is equivalent to $U(A) - U(B) \leq U(C) - U(D)$. Such a utility function is determined if:

1.) $P(A,B) + P(B,A) = 1$,

2.) $P(A,B) \leq P(C,D)$ $\quad P(A,C) \leq P(B,D)$, and[3]

3.) if $P(B,A) \leq q \leq P(C,A)$, then there is a D such that $P(D,A) = q$.

The present study will use a special form of probabilistic choice which is consistent with this stochastic axiomatization.

For simplicity of estimation, the choice model used in this experiment assumes that probability of error is a function only of the difference in expected utility between the two lotteries. The model allows for phenomena such as "looking out the window" only to the extent that they are functions of difference in expected utility.

Experimental applications of the model require data involving choices among a number of lotteries. When an individual is confronted with a pair of lotteries X and Y, the probability of choosing X is a function of the expected utility difference between X and Y. More formally:

$$(1) \qquad P(X,Y) = \Phi[U(X) - U(Y)] = \Phi[EUD]$$

For convenience, the argument of Φ is referred to as EUD (expected utility difference).

Each lottery will consist typically of several payoffs, each with an associated probability. In the experiment as a whole, there will be n different payoffs, $z_1, \ldots, z_n$; and each payoff will appear in a number of lotteries. Thus (1) can be written as:

---

1. Following Debreu, op. cit.
2. Read "the probability of choosing A when the pair A,B is offered," etc.
3. Axiom 2 is suggested by the following chain of reasoning:

$$P(A,B) \leq P(C,D) \rightleftarrows U(A) - U(B) \leq U(C) - U(D) \rightleftarrows U(A) - U(C) \leq U(B) - U(D) \rightleftarrows P(A,C) \leq P(A,C)$$

$$(1a) \qquad P(X,Y) = \Phi\left[\sum_{j=1}^{n} p_j U(z_j) - \sum_{j=1}^{n} q_j U(z_j)\right]$$

where p_j is the probability of winning the j^{th} payoff on lottery X and q_j is the probability of winning the j^{th} payoff on lottery Y. Since only a few of the n payoffs will appear in lottery X, many of the p_j's will be zero. Similarly, many of the q_j's will be zero, since only a few payoffs will appear in lottery Y.

A reader accustomed to Neumann-Morgenstern utility may be puzzled by (1) and (1a) since EUD (the argument of Φ) is not determined if an arbitrary selection of origin and unit measurement can be made. While the value of EUD will not be affected by changes in origin, it will be affected by changes in the unit of measurement. To preserve probabilities of choice, Φ[EUD], the utility assignments, $U(z_1), \ldots, U(z_n)$, must be determined to an additive constant, i.e., only the origin can be selected arbitrarily.

The reader may, if he prefers, regard the utility assignments to the various payoffs as determined only to a linear transformation. Any new utility assignments, $V(z_1), \ldots, V(z_n)$, obtained by an arbitrary selection of the unit of measurement, must, however, belong to the same Neumann-Morgenstern family as the original assignment, $U(z_1), \ldots, U(z_n)$. If the utility assignments are regarded to be unique only to a linear transformation, then each $U(z_j)$ in (1a) should be replaced by a product of the corresponding utility assignment $V(z_j)$ and an additional parameter of individual behavior k—such that the probabilities, Φ[EUD], are unaffected by arbitrary changes in the unit of measurement. Given any specific utility assignments, $V(z_1), \ldots, V(z_n)$, the value of k must be selected so that $[kV(z_1)] = U(z_1), \ldots, [kV(z_n)] = U(z_n)$. Thus the utility function, i.e. $V(z_1), \ldots, V(z_n)$, is determined only up to a linear transformation, yet P(X,Y), a function of $[kV(z_1)], \ldots, [kV(z_n)]$, has a unique determination. While it is of no consequence whether the utility parameters of (1) and (1a) are regarded as $U(z_j)$'s or $[kV(z_j)]$'s, for convenience the $U(z_j)$ convention is used in this study.[1]

The basic assumption of the choice model is that an individual stochastically maximizes expected utility. Consequently, the function Φ should posess the following properties:

1.) $\Phi(EUD) > 0$ [monotonic increasing]

2.) $\Phi(0) = \frac{1}{2}$ $[U(X) = U(Y) \rightarrow P(X,Y) = P(Y,X)]$

3.) $\Phi(+\infty) = 1$, $\Phi(-\infty) = 0$ $[0 \leq P(X,Y) \leq 1]$

4.) $\Phi(EUD) + \Phi(-EUD) = 1$ $[P(X,Y) + P(Y,X) = 1]$

1. See section 2.4 for additional discussion of the uniqueness of the utility estimates.

The function used in this study is sketched in Figure 2.1.

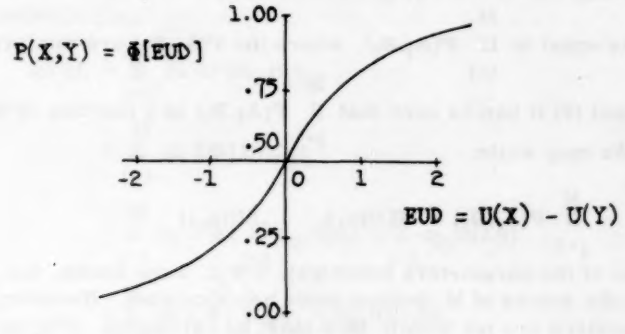


Figure 2.1

The explicit form of the function:

$$(2) \qquad P(X,Y) = \begin{cases} \frac{1}{2} \exp [U(X) - U(Y)], & \text{if } U(X) \leq U(Y) \\ 1 - \frac{1}{2} \exp - [U(X) - U(Y)], & \text{if } U(X) \geq U(Y) \end{cases}$$

U(X) denotes the expected utility of lottery X and
U(Y) denotes the expected utility of lottery Y.

was selected for three reasons. First, it satisfies properties 1 to 4. Second, it is consistent with some empirical results of Mosteller and Nogee. This consistency can be seen by comparing Figure 2.1 with Mosteller and Nogee's Figures 4a–4d,[1] in which they plot (for a number of subjects) the percent of correct predictions as a function of expected utility difference. Third, it leads to a relatively convenient computational problem.

2.2 The Estimation Procedure

This stochastic model provides a basis for a maximum likelihood estimation procedure. An individual's choices between lottery pairs provide data; the parameters to be estimated are the utilities of the various payoffs.

Suppose an individual (who behaves according to the model) makes choices between M lottery pairs involving n payoffs. The probability of a particular sequence of M choices is the product of the probabilities of the

1. Mosteller and Nogee, op. cit., pp. 390–92.

various choices. Let A_i represent the chosen lottery and B_i represent the rejected lottery for the i^{th} choice. Then, the probability of the sequence of M choices is equal to $\prod_{i=1}^{M} P(A_i,B_i)$, where the $P(A_i,B_i)$ are the form given in (2).

From (1a) and (2) it can be seen that $\prod_{j=1}^{M} P(A_i,B_i)$ is a function of the various utilities. We may write:

$$(3) \qquad \prod_{j=1}^{M} P(A_i,B_i) = \Psi[U(z_1), \ldots, U(z_n)]$$

If the values of the parameters (utilities), $U(z_j)$, were known, the probability of this specific series of M choices could be calculated. However, the values of the parameters are not known; they must be estimated. For any specific "trial" utility assignments, a "likelihood" (L) of the sample of choices[1] can be computed:

$$(3a) \qquad L = \prod_{i=1}^{M} P(A_i,B_i) = \Psi[U(z_1), \ldots, U(z_n)].$$

Since the payoffs are—in this experiment—monetary payoffs, the utility estimates are restricted to indicate a non-decreasing function of money. If utility estimates are obtained which maximize L, they are maximum likelihood estimates and possess all of the desirable properties of such estimates.[2]

This maximization cannot be done directly, and it would be tedious—though possible in principle—to perform it by trial-and-error iteration. In the next section, a computational procedure which can obtain maximum likelihood estimates for (3a) from choices among lottery pairs will be developed. Given a utility assignment for each payoff, a value of (3a) is implied; it is the task of the computation procedure to find utility assignments which maximize (3a). A linear programming problem involving "piecewise linear approximations" is constructed in Section 3.3 so that maximization of the objective function will maximize (3a). Each payoff, z_j, is an activity and the level of operation of that activity is the utility assignment, $U(z_j)$. One of the payoffs can be assigned a utility arbitrarily, since expected utility differences do not depend upon the origin of the utility function.

---

1. See Alexander McFarlane Mood, Introduction to the Theory of Statistics (New York: McGraw-Hill Book Company, Inc., 1950), pp. 152–54.

2. See Maurice G. Kendall and Alan Stuart, The Advanced Theory of Statistics (New York: Hafner Publishing Company, 1961), Vol. 2, Chapter 18.

2.3 Computational Procedure

It will be convenient to maximize the logarithm of (3a); maximization of (4) will maximize (3a).

$$(4) \qquad \ln 2L = \sum_{i=1}^{M} \ln 2P(A_i,B_i)$$

$$= \sum_{i=1}^{M} \ln 2\Phi[EUD_i]$$

$$= \sum_{i=1}^{M} \ln 2\,\Phi[\sum_{j=1}^{n} p_{ij}U(z_j) - \sum_{j=1}^{n} q_{ij}U(z_j)]$$

$$= \sum_{i=1}^{M} \ln 2\,\Phi[\sum_{j=1}^{n} (p_{ij} - q_{ij})U(z_j)]$$

p_{ij} is the probability of the j^{th} payoff on the i^{th} chosen lottery, A_i, and q_{ij} is the probability of the j^{th} payoff on the i^{th} rejected lottery, B_i. Since the j^{th} payoff will appear in only a small fraction of the lotteries, many of the p_{ij}'s and q_{ij}'s will be zero. The value of (4) depends on the utility assignments, $U(z_j)$. A linear programming problem with utilities as activity levels can be constructed so that specific utility assignments cause the objective function to take a value which approximates the value of (4). Solution of the LP optimization problem leads to utility assignments which maximize (4).[1]

Since (4) is a summation over the M choices, the contribution to (4) made by each choice can be a separate consideration. The argument of Φ, EUD_i, for the i^{th} choice may be expressed as a linear combination of the assignments to the utility activities. The assignments, $U(z_j)$, are not used directly in the objective function; rather additional activities are constructed for each choice which are "restrained" to sum to the value of EUD_i. These activities measure, through their coefficients in the objective function, the contribution to (4) caused by a specific value of EUD_i—which is computed from the specific utility assignments.

---

1. Since payoffs appear in a number of lotteries and presumably appear sometimes on chosen lotteries and sometimes on rejected lotteries, an increase in the value of (4) cannot occur without limit. While an increase in some $U(z_j)$ will raise $\ln 2\,P(A_i,B_i)$ for those lottery pairs which contain $U(z_j)$ on the chosen lottery, such an increase will diminish $\ln 2\,P(A_i,B_i)$ for those lottery pairs which contain $U(z_j)$ on the rejected lottery. See Section 3 for an extended discussion of this tradeoff.

Since ln 2 $\Phi[EUD_i]$ is a non-linear function of EUD_i (see Figure 2.2), a piecewise linear approximation if employed (see Figure 2.3). If EUD_i is positive, the pieces in the first quadrant (Figure 2.3) are used in the approximation. But if EUD_i is negative, the piece in the third quadrant is used.

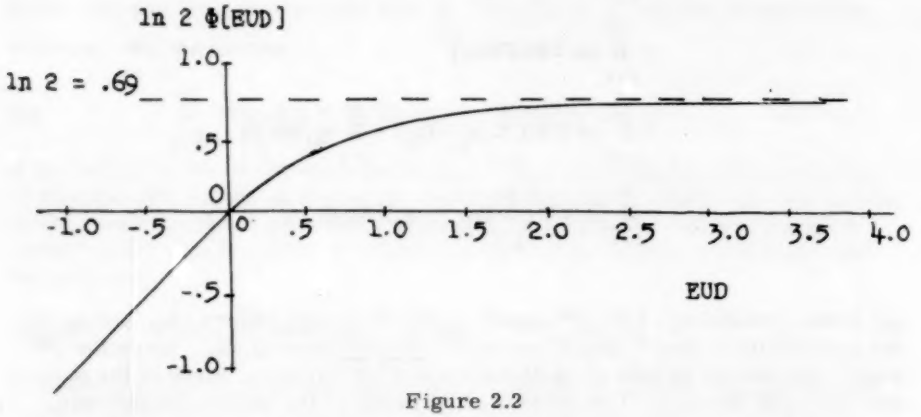


Figure 2.2

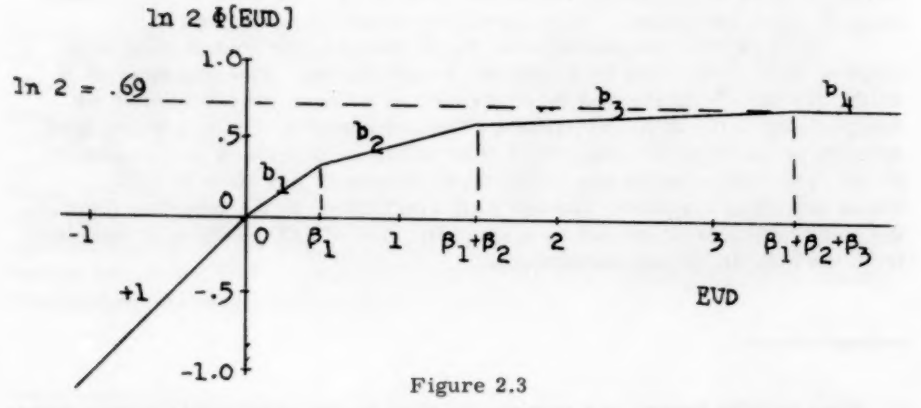


Figure 2.3

The first quadrant—positive EUD_i—contains four pieces. The slope of the ℓ^{th} piece is b_ℓ and the upper bound of the ℓ^{th} piece is β_ℓ. The length of the ℓ^{th} piece is represented in the linear programming problem by $\omega_{i\ell}$, and $b_\ell \omega_{i\ell}$ is the contribution of the ℓ^{th} piece to ln 2 $\Phi[EUD_i]$. Consequently—if EUD_i is positive—$\sum_{\ell=1}^{4} \omega_{i\ell}$ will take the value of EUD_i and $\sum_{\ell=1}^{4} b_\ell \omega_{i\ell}$

will approximate the value of ln 2 $\Phi[EUD_i]$. The piece in the third quadrant—negative EUD_i—has the slope of +1. This piece has a value equal to $-\lambda_i$ and ln 2 $\Phi[EUD_i]$ will equal $1(-\lambda_i)$ or $-1\lambda_i$.

This approximation is summarized in Table 1, which shows the relation between EUD_i, the values of the approximation activities, and ln 2 $\Phi[EUD_i]$. The piecewise linear programming problem which results is summarized in (5). Of course, all variables are restricted to be non-negative. The problem is also shown in a tableau (Table 2). In addition to (5), nine restraints to guarantee monotonicity are included in the tableau.[1]

$$(5)\quad \max \sum_{i=1}^{M} (-1\lambda_i + b_1\omega_{i1} + b_2\omega_{i2} + b_3\omega_{i3} + b_4\omega_{i4}) \quad \text{[Objective Function]}$$

$$\sum_{j=1}^{n} (p_{ij} - q_{ij})U(z_j) + \lambda_i - \omega_{i1} - \omega_{i2} - \omega_{i3} - \omega_{i4} = 0 \quad \text{[Choice Restraint]}$$

$$\omega_{i1} \le \beta_1 \quad \text{[Upper}$$
$$\omega_{i2} \le \beta_2 \quad \text{Bound}$$
$$\omega_{i3} \le \beta_3 \quad \text{Restraints]}$$
$$(i = 1, \ldots, M)$$

TABLE 1

if	then					and
EUD_i	λ_i	ω_{i1}	ω_{i2}	ω_{i3}	ω_{i4}	Value of Objective Function
Below 0	$-EUD_i$	0	0	0	0	$-1(-EUD_i)$
$0 - \beta_1$	0	EUD_i	0	0	0	$b_1 EUD_i$
$\beta_1 - \beta_1 + \beta_2$	0	β_1	$EUD_i - \beta_1$	0	0	$b_1\beta_1 + b_2[EUD_i - \beta_1]$
$\beta_1 + \beta_2 - \beta_1 + \beta_2 + \beta_3$	0	β_1	β_2	$EUD_i - (\beta_1 + \beta_2)$	0	$b_1\beta_1 + b_2\beta_2 + b_3[EUD_i - (\beta_1 + \beta_2)]$
$\beta_1 + \beta_2 + \beta_3$ Above	0	β_1	β_2	β_3	$EUD_i - (\beta_1 + \beta_2 + \beta_3)$	$b_1\beta_1 + b_2\beta_2 + b_3\beta_3 + b_4[EUD_i - (\beta_1\beta_2 + \beta_3)]$

1. The b_ℓ and β_ℓ are shown in the tableau. The approximation is accurate to .015, i.e., the error of $P(A_i, B_i)$—as a function of EUD_i—does not exceed .015. In most ranges of EUD_i, the approximation is more accurate.

In the piecewise linear approximation constructed above (see Table 1) it was presumed that:

1. λ_i, or $\sum_{\ell=1}^{4} \omega_{i\ell}$, or both equal zero for all i, and

2. $\omega_{i1} < \beta_1 \rightarrow \omega_{i2} = 0$, and $\omega_{i2} < \beta_2 \rightarrow \omega_{i3} = 0$, and $\omega_{i3} < \beta_3 \rightarrow \omega_{i4} = 0$.

$\omega_{i3} = 0$ $\omega_{i4} = 0$

$\omega_{i4} = 0$

If $\lambda_i > 0$ and $\sum_{\ell=1}^{4} \omega_{i\ell} > 0$, an equal reduction of both, until one reaches zero, would not affect the utility assignments but would increase the value of the objective function. This is true, since the coefficient in the objective function for λ_i is -1 and the coefficient in the objective function for $\omega_{i\ell}$ for each ℓ is less than 1 (see Figure 2.3). Similar reasoning suggests that, since $b_1 > b_2 > b_3 > b_4$, it will always pay to introduce ω_{i1}, ω_{i2}, ω_{i3}, ω_{i4}, in that order.

In summary, "trial" utility assignments imply an expected utility difference for each of the M choices. λ_1 or $\sum_{\ell=1}^{4} \omega_{i\ell}$ will be set equal to $|EUD_i|$ and the contribution to maximization will occur through the coefficients of λ_i or $\omega_{i\ell}$ in the objective function. The search for an optimal solution to this linear programming problem is a search for utility assignments for the various payoffs which determine values of λ_i and $\omega_{i\ell}$, such that the objective function (see tableau) is a maximum. Typically, an iteration will involve the substitution of an $\omega_{i\ell}$ or λ_i for another $\omega_{i\ell}$ or λ_i in the basic solution. Such a change will cause a change in the utility assignments and a change in the value of the objective function. As activity $\omega_{i\ell}$ replaces λ_i in the basis, EUD_i may switch from negative to positive. When no further change in the basis—i.e., change in utility assignments—can be made to increase the value of the objective function, the process stops, and the utility assignments which obtain are maximum likelihood estimates.

2.4 Uniqueness of the Estimates[1]

Part 1.4 asserted that the estimation of procedure of this study did not require artificial restrictions or "normalizations" (e.g. utility intervals no less than +1). Such restrictions are required with the Davidson-Suppes and the Manne criteria to avoid the trivial result—all payoffs with equal utility

1. This discussion extends the remarks made in Part 2.1 concerning the uniqueness of the utility assignments used in our stochastic model of choice.

TABLE 2

Summary of Linear Approximation of $\ln 2\Phi[\mathrm{EUD}]$

Levels of Operation		Payoffs $U(z_1) \ldots U(z_{10})$	Approximation Activities $\lambda_1 \ldots \lambda_{50}$	$\omega_{1,1} \ldots \omega_{50,1}$	$\omega_{1,2} \ldots \omega_{50,2}$	$\omega_{1,3} \ldots \omega_{50,3}$	$\omega_{1,4} \ldots \omega_{50,4}$	
Obj. Func.		0 ... 0	−1 ... −1	.720 ... .720	.256 ... .256	.055 ... .055	0 ... 0	
Choice Restraints	1	$(p_{ij} - q_{ij})$	1 0	−1 0	−1 0	−1 0	−1 0	= 0
	50		0 1	0 −1	0 −1	0 −1	0 −1	= 0
Upper Bound Restraints	51	0	0	1 0	0	0	0	≤ .45
	100			0 1				≤ .45
	101	0	0	0	1 0	0	0	≤ 1.00
	150				0 1			≤ 1.00
	151	0	0	0	0	1 0	0	≤ 2.05
	200					0 1		≤ 2.05
Monotonicity Restraints	201	−1 +1 0 / −1 +1	0	0	0	0	0	≥ 0
	209	0 −1 +1						≥ 0

assignments. In the present study, the so-called trivial result would mean (since EUD's would equal zero) a likelihood of $(1/2)^M$. This provides a lower bound on the value of the likelihood function, and it is likely that alternative utility assignments can be found which imply a higher valued likelihood function. (Many choices would have likelihoods greater than 1/2 and only a few less than 1/2.)

This difference among the estimation techniques arises because utility functions in this study contain more information than the traditional Neumann-Morgenstern utility function (see Part 2.1). As a pragmatic matter, Neumann-Morgenstern utility is used to order lotteries in terms of expected utility. Such a ranking can be preserved by a family of utility functions related by linear transformations. Since the Davidson-Suppes and the Manne criteria were intended to measure "Neumann-Morgenstern" utility, all members of a "family" of utility functions related by linear transformation should be judged equivalent in terms of consistency. The estimates from this study, however, not only rank lotteries but also convey information about the probabilities of various choices (see Part 2.1). In our stochastic model the probability of choosing one lottery over another for an individual is a function of the utilities of the payoffs in the two lotteries. Such probabilities are preserved only if the "family" of utility functions is restricted to those related by additive constants. Thus an artificial restriction such as utility intervals no less than +1 is not required nor is it legitimate. (But perhaps it is justifiable—on grounds of rationality—to require the utility function to be non-decreasing.)

The justification for utility "unique to an additive constant" can be illustrated by an example. Consider three utility functions which are linear in money but differ in slope. From each of these functions, expected utility differences can be calculated for an individual's choices between lottery pairs. If the slope of function 2 is twice the slope of function 1, and the slope of function 3 is three times the slope of function 1, the EUD's computed from function 2 will be twice as large (in absolute value) as the EUD's computed from function 1, and the EUD's from function 3, three times as large. Perhaps the following numbers might occur.

Choice Number	EUD (1)	EUD (2)	EUD (3)
1.	+3/4	+1 1/2	+2 1/4
2.	+1	+2	+3
3.	-1/4	-1/2	-3/4
4.	0	0	0
5.	+1/4	+1/2	+3/4

Although these three functions give equivalent results for the ranking of lotteries (since they are related by linear transformation), it can be seen that

the results are not equivalent in the context of the choice model and estimation procedure discussed in this section. From the expected utility differences, a value of the objective function can be calculated.

Choice Number	EUD (1)	Contribution to Objective Function	EUD (2)	Contribution to Objective Function	EUD (3)	Contribution to Objective Function
1.	+3/4	+.40080	+1 1/2	+.58275	+2 1/4	+.62400
2.	+1	+.46480	+2	+.61025	+3	+.66525
3.	−1/4	−.25000	−1/2	−.50000	−3/4	−.75000
4.	0	0	0	0	0	0
5.	+1/4	+.18000	+1/2	+.33680	+3/4	+.40080
Value of Objective Function		+.79560		+1.02980		+.94005

The second linear function leads to a higher value of the likelihood function than the first or third functions; consequently, the second is preferred. Although all imply the same ranking of lotteries, the three functions differ in estimates of probabilities for the various choices.

In general, from a family of utility functions unique to a linear transformation, one subset of the family (unique to an additive constant) will maximize the likelihood function. Just as the selection of slope in the family of linear utility functions has been shown to be of some consequence; so, too, will the selection of slope be important in other families of utility functions related by linear transformation.

In addition to providing discrimination within families related by linear transformation the choice model and estimation procedure provide discrimination between families where each family is represented by its "maximum likelihood member." The value of the likelihood function depends not upon ranking of lotteries (Neumann-Morgenstern utility) but upon differences in expected utility between accepted and rejected lotteries. Estimates which preserve probabilities of the various choices (and thus the value of the likelihood function) also preserve the ranking of lotteries, but estimates which preserve the ranking of lotteries do not, in general, preserve probabilities of the various choices. In earlier estimation techniques the ranking of lotteries was regarded as the primary result, but in this formulation the ranking of lotteries is an (important) secondary result.

3. Experimental Design

3.1 General Problems

This section is concerned with the selection of payoffs and lottery pairs for the experimental part of the study. Choice data were collected

both for estimation and for prediction. The first part discusses problems in the selection of estimation lottery pairs; the second discusses the actual selection of payoffs and lottery pairs; and the third describes a theoretical pilot study.

In choosing payoffs and lottery pairs, the experimenter must consider several problems and limitations. First, the number of estimation lottery pairs is limited by computation facilities. The piecewise linear approximation (see Part 2.3) requires three upper bound restraints—in addition to the choice restraint—for each lottery pair. For this study, a 200-restraint linear programming problem (50 estimation lottery pairs) for each of 10 subjects seemed to exhaust the amount of computer time available.[1]

Second, only a limited number of choices can be obtained from a subject. At some point the subject will become bored; consequently the value of additional choices will be questionable.[2] Although the "boredom effect" is difficult to assess, the effective restraint on the number of estimation lottery pairs seemed to be the computation limitation. Of course a judgment had to be made that a sufficient number of prediction lottery pairs could also be included without encountering boredom effects.

Given a limited number of estimation lottery pairs, what pairs should be included? How many different payoffs should be used in the experiment? If the form of the utility function cannot be specified, then each payoff provides an observation on the utility function. However, fifty estimation lottery pairs permit only 200 payoff appearances (four in each lottery pair). An increase in the number of payoffs can be obtained only through a decrease in the average number of appearances per payoff. The effect of this decrease is equivalent to loss of "degrees of freedom" in standard statistical problems. That is, estimates may be obtained which "explain" the sample data accurately, but they will give poor prediction on independent data.

Thus, a tradeoff exists between the number of observations on the utility function (payoffs) and the number of appearances by each payoff. I know of no criteria to apply to this tradeoff. Consequently the decision to include ten payoffs with 20 appearances per payoff was made on the basis of previous experience (i.e., the Manne experiment) and intuition. This decision was not independent of some considerations discussed in the next part.

Given the number of different payoffs, their values and their pairings must be decided. Poor selection of lottery pairs can give rise to difficult estimation problems. Suppose a particular payoff, which appears in a number of lottery pairs, is either 1.) always chosen (i.e., is never on a rejected lot-

---

1. Nine additional restraints were added because the utility assignments were restrained to be non-decreasing (see Part 2.3).

2. Paul Slovic, Sarah Lichtenstein, and Ward Edwards, "Boredom-Induced Changes in Preferences among Bets," Unpublished paper.

tery), or 2.) always rejected (i.e., never on a chosen lottery). Since the likelihood expression (Part 2.2) is a function of expected utility differences, maximization of this expression will involve a utility assignment of plus infinity to a payoff which is never rejected (case 1) and a utility of minus infinity to a payoff which is never chosen (case 2). This assignment results in an expected utility difference of infinity and consequently a likelihood of one in all of the lottery pairs in which the never-rejected (or never-accepted) payoff appears. This result is, of course, intuitively clear. The individual in case 1 has provided through his choices no suggestion that the never-rejected payoff might be less desirable than any other payoff or combinations of other payoffs. Similarly, case 2, the individual offers no indication that the never-chosen payoff might be as desirable as any other payoff.

It is feasible to select the lottery pairs so that a specific payoff would seldom be either "always chosen" or "always rejected." But the problem is not simply "black and white." Suppose two payoffs which are relatively close in "true" utility appear in lottery pairs such that the higher payoff is more frequently on the rejected lottery. The relative frequency of rejections of the higher payoff may result in a lower utility estimate for the higher payoff than for the lower payoff.[1]

Of course, the direction of bias cannot be determined merely by calculating for any payoff the ratio of number of times chosen to number of times rejected. Since the likelihood for any choice depends upon expected utility difference, the effect on the likelihood function of a marginal change in a utility assigned to a specific payoff is not independent of the utilities assigned to the other payoffs. For example, suppose a specific payoff is frequently rejected in lottery pairs where "true" differences in expected utility are large, and is chosen in a few lottery pairs where "true" expected utility differences are small. The net effect might be to over-estimate, not under-estimate, the utility of that payoff;—a high estimate will increase the value of the likelihood function. This difficulty can be reduced by increasing the number of times a given payoff appears, so that it appears in a variety of combinations with other payoffs.

3.2 Payoffs and Lottery Pairs

Most of the earlier experiments involving real money contained low stakes (usually under $1). For payoffs of this size, it can be argued that most individuals find the game uninteresting at least in terms of financial rewards. Thus for this study the maximum payoff was set at $9.75.

---

1. Of course, if the non-decreasing restriction suggested in Section 2 is applied, the "perverse" estimates cannot occur. (Consequently, in the never-rejected and the never-accepted cases, only the highest and lowest payoffs respectively could be assigned infinity.)

In addition to low payoffs, most earlier laboratory experiments involved odd payoffs, such as $.62, $.17, and −$.06; however, in this experiment, payoffs can be expressed in 25 c. units. It was thought that payoffs expressed in "round numbers" would appear more realistic to subjects. In fact, at the end of the last session, all subjects denied that they had rounded payoffs (e.g., thought of $9.75 as $10.00).

In the "Manne experiment," subjects showed very marked risk aversion to negative payoffs, so two negative payoffs were included to obtain evidence on this part of the utility function. The payoffs selected were −$1.00 and −$1.50.

It was expected that most subjects would be risk-averters. Therefore, on the average, intervals between payoffs were smaller at the lower end of the scale. Thus, many lottery pairs could be constructed which, for these subjects, would have small expected utility differences. Such lottery pairs result in variation in choices among subjects and, thus, permit discrimination among the utility functions of different subjects. The payoffs $0, $.50, $1.25, $2.75, $4.00, $6.50, and $8.25 were selected to complete the list of ten.

To avoid subjective probability distortion, the estimation lottery pairs were restricted to lotteries with probability 1/2 associated with each payoff. Given this restriction, only one non-trivial choice can be formed, i.e., only one which could not be predicted with the knowledge that the utility function is monotonic increasing. This is a lottery containing the highest and lowest payoffs against one containing the two intermediate payoffs. Thus from 10 payoffs, $\binom{10}{4}$ or 210 non-trivial lottery pairs can be obtained. Repetition of lottery pairs was avoided because of the possibility that first choices would be remembered.

Fifty estimation lottery pairs were selected from the 210 possible lottery pairs in the following way. The 210 lottery pairs were ranked according to difference in expected value between high-variance lottery and low-variance lottery. Then these lottery pairs were split into five groups of 42 according to difference in expected value, and ten lottery pairs from each of these five groups were selected to be estimation lottery pairs. An attempt was made to select the 10 lottery pairs from each group so as to reproduce the relative frequencies of the payoffs in that group. In addition, relative frequencies (of the payoffs) on high-variance and low-variance lotteries within each group were preserved. This procedure was designed to minimize the bias, discussed in Part 3.1, arising from giving any one payoff a disproportionate number of favored positions.

The utility estimates are tested, in part, by their ability to predict choices—relative to other prediction techniques. Discrimination between prediction techniques will not be statistically significant unless a large number of prediction discrepancies are obtained (see Section 5). Consequently, 250 prediction lottery pairs were formulated in addition to the 50 estimation

lottery pairs. These prediction lottery pairs were varied in a number of dimensions so that a broad comparison of techniques could be obtained. Prediction lottery pairs were constructed with three probability combinations—100 pairs with 1/2−1/2, 100 with 2/3−1/3, and 50 with 9/10−1/10. Within each probability type, lotteries were varied with respect to expected value and risk. There are a large number of possible lottery pairs which can be formed from these probabilities and 10 payoffs. However, many of these appear to be obvious choices; consequently, some of them were dropped immediately, thus making the selection problem more tractable.

3.3 A "Monte Carlo" Experiment

Suppose the decision process of an individual can be represented by the choice model described in Part 2.1. This implies that the individual behaves as if he: 1.) calculates expected utility differences, 2.) computes the probability of selecting one of the lotteries, and 3.) consults a random number table (it's faster than flipping coins) to determine which lottery he selects. If this individual were requested to make a series of choices between lotteries, experimental data would be obtained which could be used to estimate that individual's utility function with the estimation procedure described in Part 2.3.

The resulting estimates would not be expected to lie exactly on his "true" utility function for two reasons. First, estimates will have a "sampling distribution." Since the model involves a stochastic process, estimates of the "utility parameters" will exhibit random deviations from their "true" values. Of course, the distribution depends, in part, upon the experimenter's selection of the specific payoff combinations, i.e., lottery pairs. Second, errors may result because piecewise linear programming is only an approximation to maximum likelihood estimation. To get some notion as to the errors of utility estimates caused by these factors in the aggregate, a "Monte Carlo" pilot study was run. "Imaginary" subjects were assigned utility functions, and, subsequently, these "subjects" made choices according to the choice model.

For this study, two arbitrary utility functions were specified. One (suggested by Pfanzagl[1]) takes the form $U(x) = A\ \lambda^x + B$, where $A < 0$, $0 < \lambda < 1$. This function for $A = -25.7$, $B = 29.9$, $\lambda = 1/e$, and x measured in \$10 units is displayed in Figure 3.1.[2] This function possesses two properties which make it a desirable example: 1.) it suggests risk aversion and 2.) ranking of lotteries is independent of changes in wealth.

---

1. Johann Pfanzagl, "A General Theory of Measurement—Applications to Utility," Naval Research Logistics Quarterly, Vol. 6 (1959), pp. 290–91.

2. Actual estimates are reported in Appendix A. For purposes of visual comparison the utility estimates were subjected to linear transformation so that U(\$9.75) = 50 and U(\$0) = 0 (see Figures 3.1, 3.2 and 3.3).

The other function (displayed in Figure 3.2) is a free-hand construction which indicates extreme risk aversion for negative payoffs (relative to the function above), a region of increasing marginal utility from \$0 to \$4.00, and risk aversion above \$4.00. Examination of the results from the Manne experiment and armchair reflection suggested this utility function might be representative of a number of individuals.

Given these functions, choices were generated in the manner described in the first paragraph of this part. That is, all of the requirements of the choice model (Part 2.1) were met, so that estimation errors would in no way reflect failure to correctly duplicate the rules of behavior of the "subject." The utilities were selected so that the expected number of "errors" for the 50 choices was approximately seven. In fact, six "errors" were made from the function displayed in Figure 3.1 and seven "errors" from Figure 3.2. None of these "errors" (inconsistent choices) had a probability of less than one-tenth.

Estimates of the utility function were obtained by three methods: maximum likelihood, Davidson-Suppes minimum Θ, and Manne minimum $\Sigma\Theta$ (see Section 1). From the generated data, utility assignments could be found which suggest no inconsistent choices; therefore the Davidson-Suppes and the Manne criteria gave identical estimates. As can be seen from Figures 3.1 and 3.2, maximum likelihood estimation gives a closer approximation to both functions. In the first function (Figure 3.1), the superiority of maximum likelihood is immediately apparent; however, the distinction is less marked in the second function (Figure 3.2).

From Figure 3.1, it can be seen that the Davidson-Suppes and Manne criteria resulted in equal utility assignments to \$9.75 and \$8.25; to \$4.00 and \$2.75; to \$1.25, \$.50, and \$0; and to $-$\$1.00 and $-$\$1.50. Frequently, equal

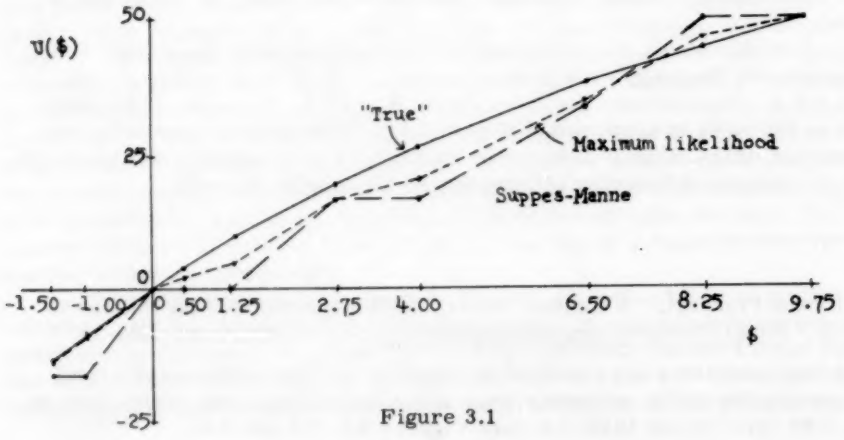


Figure 3.1

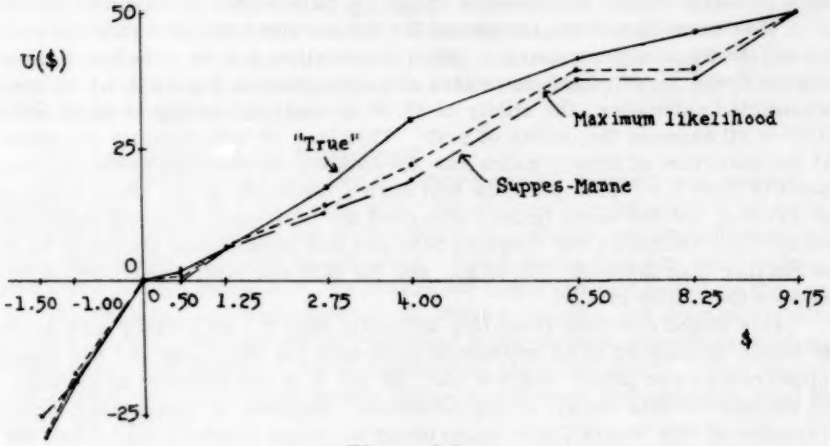


Figure 3.2

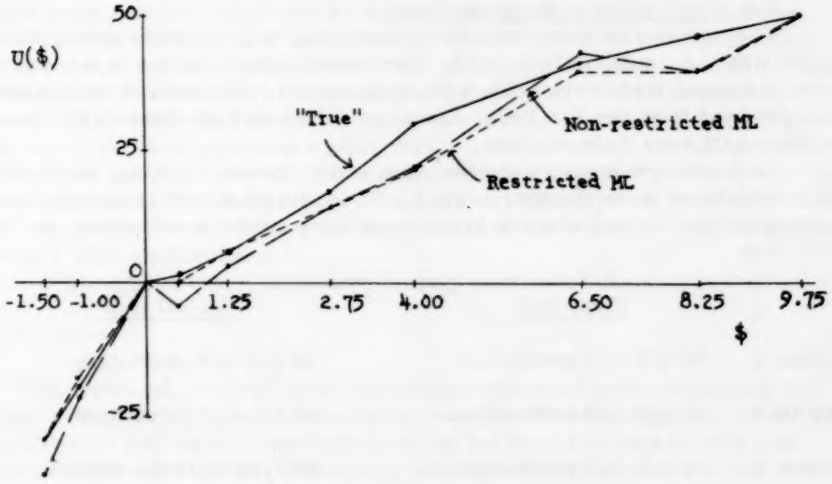


Figure 3.3

utility assignments were obtained for the two techniques in the Manne et al. experiment. These phenomena exhibit the consequences of the failure of either the Davidson-Suppes or the Manne criteria to reward, in choosing among possible utility assignments, positive differences in expected utility.

Maximum likelihood estimates for the second function had to be restrained to be non-decreasing. (Such restriction led to a better fit; the restricted and unrestricted estimates are compared in Figure 3.3.) In the unrestricted estimates, the utility of \$6.50 exceeds the utility of \$8.25 and the utility of \$0 exceeds the utility of \$.50. This lack of monotonicity suggests that the selection of lottery pairs has not entirely avoided the difficulty discussed in Part 3.1. It is possible that such a difficulty would arise from random errors, but the same failure occurred for a number of actual subjects. Unrestricted estimates for subjects SO3 and SO8 implied the utility of \$6.50 was greater than the utility of \$8.25, and for SO3 and SO4, the utility of \$0 exceeded the utility of \$.50.

One might conclude from this difficulty that 1.) the lottery pair selection should be juggled in an attempt to eliminate the difficulty, 2.) the number of appearances per payoff was too low, or 3.) it is not possible to provide good estimation data for all utility functions. Because of computation delays, the results of this Monte Carlo experiment were not obtained until after the experimental data had been collected. Thus, it was not possible to alter the structure of lottery pairs. However, since estimates were restrained to be non-decreasing, at least part of the problem was avoided.

4. The Experiment

4.1 Description of the Experiment

The experiment was run in three sessions, held two days apart, during the week, June 25–July 1, 1962. Ten subjects participated in the experiment; individual tests were made with each subject. One-half of the subjects were obtained from the Air Force Language School at Yale University, and the other half were Yale students.

Each session required approximately 100 choices. Lottery pairs with 1/2 probabilities were always run first. The cells of the following table indicate probabilities and whether the choices were used for estimation or prediction.

	First Half	Second Half
Session 1	50 1/2-1/2 prediction	50 2/3-1/3 prediction
Session 2	50 1/2-1/2 estimation	50 2/3-1/3 prediction
Session 3	50 1/2-1/2 prediction	50 9/10-1/10 prediction

In addition to the 300 lottery pairs for estimation and prediction, the subjects

were confronted with 15 dominated (or trivial) lottery pairs. These choices were dominated in the sense that a choice could be predicted with the assumption that the utility function is non-decreasing (see Part 2.1). Such choices are taken to be non-stochastic in the model used in this study. Choices are assumed to be stochastic only when an individual must evaluate the size of the difference in utility between various payoffs, not merely the sign of these differences. In all cases subjects chose the correct lottery, i.e., the lottery with the higher expected utility.

4.2 Comments on the Experiment

Determination of the number of lotteries to be "played" and when to play them was an issue. Theoretically, the choices are intended to provide independent observations on the utility function. If only one choice were to be played, this objective might be achieved. But if more than one were to be played and a subject knew this, he should not make choices based on the expected utility of one lottery. Rather, he should consider the probability distributions of total winnings which can occur from various sequences of choices and then select the sequence which maximizes expected utility. That is, he should regard the experiment as one composite lottery with possible total winnings as the payoffs.

It is undesirable to change the wealth of a subject between sessions (which might encourage a change in behavior). However, to maintain a feeling of realism for each session, perhaps subjects should play one lottery at the conclusion of each session. A compromise between these two considerations was required. It was decided that a subject should draw at random a lottery pair at the conclusion of each session, but the lottery which he had chosen from that pair would not be played until the end of the last session. The draw of the lottery pair added realism; however, it was felt that a subject would not feel his wealth position had markedly changed. No lottery had a guaranteed loss and most had a low guaranteed win. It was unavoidable, however, that subjects would have some new feelings about their wealth positions vis-a-vis this experiment after they won the privilege (or obligation) to play later a specific lottery. While some subjects said this affected their behavior, this effect did not seem to be a serious difficulty (see Section 5). At least the problem would have been more serious had subjects actually played lotteries after each session.[1]

Although the above procedure largely eliminated changes in wealth

---

1. This argument, coupled with impressions obtained from interviews with subjects (that they did not require daily rewards to obtain realistic incentives), suggests that playing only one lottery would not result in appreciably less realistic choice situations. Since this would permit an increase in the size of the payoffs, observations over a wider range of the utility function could be obtained.

originating within the experiment, it is conceivable that a subject's wealth was substantially affected by external windfall gains or losses. To the extent that a subject retained a separate "account" for this experiment, such external events would not affect choices over the three sessions. Casual empiricism suggests for example that this is true of continuing poker games. Also Mosteller and Nogee's results from an experiment which took place over several months suggest that outside changes in wealth are not crucial.

It is surprising that many people consider independently each confrontation with risk. An individual has the opportunity to play "lotteries" with small payoffs each day, since he lives in a world of uncertainty. For example he may, daily, buy (or fail to buy) insurance against a parking ticket by putting (or not putting) a dime in the meter. Since the money won (or lost) from playing various daily "lotteries" goes into (or comes out of) the same pocket, it would seem as though the individual should take a "risk position" over the aggregate of these "lotteries."[1]

One of the surprising parts of administration of the experiment was the fact that subjects very rapidly made choices. Fifty choices required as much as 13 minutes, but several subjects on the last session (1/2-1/2 probability) completed 50 choices in three minutes. The Air Force subjects were on the average several minutes faster than the graduate students.

As subjects became more familiar with the process, the time required to make a choice tended to decrease. To the extent that this decrease reflected an increase in efficiency, it is not worrisome. However, it might reflect the fact that the repetition of similar choices made it useful for a subject to adopt a simple decision rule for convenience. It may be that his behavior is predicted well because he has used a consistent decision rule. Perhaps at another time in a slightly different situation, he will not adopt such a decision rule. In fact, several subjects claimed they had simple rules, although they were unable to articulate their precise nature. However, they also claimed they made each choice responsibly "as if it was the one which would determine how much he won." Thus no subject admitted that he adopted a rule for convenience which did not always represent his preference.

---

1. Some discussion of the changes in the "utility function" caused by the attainment of "payoffs" is provided by Milton Friedman and L. J. Savage, "The Utility Analysis of Choices Involving Risk," Journal of Political Economy, Vol. 56 (1948), pp. 279–304, and Harry Markowitz, "The Utility of Wealth," Journal of Political Economy, Vol. 60 (1952), pp. 151–58. Friedman and Savage presume changes in wealth move an individual along a stable utility function, but Markowitz argues the function will shift with changes in "customary" wealth or income.

5. Evaluation of the Experimental Results

5.1 Introduction

The experiment provides two types of evidence pertaining to the expected utility choice model. First, the accuracy of prediction of choices by a number of techniques (i.e., expected utility maximization, actuarial maximization, and minimax loss) can be compared. For example, consider a choice between two lotteries:

	X			Y
2/3	\$.50	and	2/3	\$6.50
1/3	\$8.25		1/3	\$0

1.) Actuarial predicts Y, since 2/3 (\$.50) + 1/3 (\$8.25) < 2/3 (\$6.50) + 1/3 (\$0).

2.) Minimax predicts X, since the lowest payoff, \$0, appears in Y.

3.) Expected Utility predicts X if:

$$2/3\ U(\$.50) + 1/3\ U(\$8.25) > 2/3\ U(\$6.50) + 1/3\ U(\$0)$$

and predicts Y if:

$$2/3\ U(\$.50) + 1/3\ U(\$8.25) < 2/3\ U(\$6.50) + 1/3\ U(\$0).$$

Second, a subject's verbal comments can be compared with the estimates of his utility function.

The experimental results are reported in this section. In Part 5.2, general tests of the stochastic expected utility hypothesis (see Part 2.1) will be discussed, and in Part 5.3 results for individual subjects are presented.

5.2 Tests in the Aggregate

The practical value of utility estimates for an individual is to be found in the predictive accuracy of expected utility maximization relative to alternative predictive techniques. Prediction based upon the "maximum likelihood" utility function should be compared with prediction based upon alternative rules of behavior such as actuarial maximization or minimax loss. A test of the hypothesis that expected utility maximization gives better prediction for an individual than an alternative rule, e.g., actuarial maximization, is described below.

For each subject, a number of lottery pairs will be predicted. Suppose, for any given lottery pair, both techniques predict the same choice. Then, there is no basis for discrimination between the two techniques. However, consider those lottery pairs for which predictions are different. If the two techniques have equal predictive ability, the conditional probability of a correct prediction by expected utility given a discrepancy would be one-half. Thus the number of correct predictions in discrepancy lottery pairs (see

Tables 3 and 4), should be distributed binomially.[1] A one-tail test of the null hypothesis that both techniques have equal predictive ability is appropriate. (We give the benefit of doubt to the simple technique if it is at least as good. Thus we are not interested in the tail which suggests expected utility is worse than actuarial maximization.) If this null hypothesis is rejected, we would conclude that expected utility is more accurate than the alternative technique.[2]

The prediction test showed expected utility was better than actuarial at the 1% level[3] for four subjects, at the 5% level for a fifth, at the 6% level for a sixth, and at the 17% level for a seventh. Expected utility was better than minimax at the 1% level for 8 of 10 subjects. Actuarial maximization resulted in more correct predictions than expected utility for only two subjects while minimax loss gave more correct predictions for only one subject. A complete listing of probabilities under the null hypothesis for this test is presented in Tables 5 and 6. The data were disaggregated by session and by probability type as shown in the tables. However, the data for the tests reported above included all sessions and all probabilities (fourth row).

These prediction tests were applied individually, and consequently the results reflect acceptance of the stochastic choice model for prediction for some individuals and rejection for others. One of the pitfalls of reporting the proportion of subjects whose choice behavior was predicted significantly more accurately by the expected utility technique is that many mild successes (or mild failures) might swamp a few dismal failures (or marked successes). Thus, perhaps the proportion of correct predictions summed over all indi-

---

1. That is, $\frac{N!}{X!(N-X)!} \cdot P^X Q^{N-X}$, where N = number of lottery pairs (where discrepancies occur) and X = number of correct predictions by expected utility (where discrepancies occur). Further P = Q = 1/2 in the null hypothesis. The area in the tail of the distribution is

$$\sum_{x=X}^{N} \frac{N!}{x!(N-x)!} P^x Q^{N-x} .$$

2. This test is described in Quinn McNemar, Psychological Statistics (New York: John Wiley & Sons, Inc., 1955), pp. 55–60. It is also suggested for this type of problem by Gordon Becker, Morris DeGroot, and Jacob Marschak, "Stochastic Models of Choice Behavior," Behavioral Science, Vol. 8 (1963), pp. 53–54. The reader will notice this is a test for two correlated proportions. Since both prediction techniques predict the same choice situations, the two proportions of correct predictions will be correlated.

3. If the null hypothesis is true, results which were at least as favorable to expected utility (as those which actually obtained) would occur with frequency no greater than one in a hundred trials. Thus we reject the null hypothesis.

viduals should be tested for significance. Although this test is inaccurate because the observations are not independent, the expected utility technique appears to be significantly better (at the .1% level) than actuarial maximization or minimax loss (see Tables 5 and 6, "Total" column).

If expected utility is not significantly better than actuarial prediction for a specific subject, it is not appropriate to conclude that the model should be rejected for that subject. The following example will illustrate the point. Suppose a subject who behaves according to the model displays actuarial behavior, except for random deviations. Expected utility maximization will not predict better (or as well as?) actuarial maximization because of errors in the utility estimates. Thus, the prediction test is not a test of whether the model is appropriate for the subject; rather it is a test of the difference between two prediction techniques. It would also be desirable to test whether the model can "explain" the behavior of subjects. A likelihood ratio test could be used for this purpose. Utilities estimated from the estimation choice data can be tested on separate choice data. Since this test would require an additional linear programming run, the test was not performed.[1]

Another test is the following. If the choice model is appropriate for various subjects, it should explain differences in consistency among them. Subjects who appear most consistent on the estimation data (in the sense that likelihood of the sample choices is high relative to other subjects) should also appear most consistent on prediction data. Therefore, if subjects are ranked according to 1.) value of likelihood function and 2.) number of correct predictions, these rankings should be correlated. A rank correlation test was performed, and the result was significant at the .6% level.[2] This evidence supports the hypothesis that the behavior of subjects can be represented by our choice model.[3]

---

1. The likelihood ratio test could also be used to test the hypothesis that a subject (assumed to behave according to the model) can be represented by (say) a linear utility function. Again an additional LP run would be required.

2. See Maurice G. Kendall, Rank Correlation Methods (New York: Hafner Publishing Company, 1955), Chapters 1, 3, and 4. This test was run between likelihoods and prediction to lottery pairs with 1/2-1/2 probabilities. Since estimation lottery pairs had probabilities equal to one-half, the influence of subjective probability distortion was minimized if not eliminated.

3. It is interesting to note that the two subjects who were predicted more accurately by actuarial maximization than by expected utility maximization (although their utility estimates were far from linear) were ranked last in likelihood and in expected utility prediction. Perhaps these subjects were so inconsistent that "good" estimates could not be obtained from their limited number of choices.

TABLE 3

Expected Utility—Actuarial Comparison

.XX or .XXX = Correct Expected Utility Predictions/Total Predictions

(Y) = Total Predictions

Session	S01	S02	S03	S04	S05
Session 1	.58 (19)	.64 (14)	.45 (29)	.44 (16)	.56 (16)
Session 2	.83 (6)	.80 (5)	.15 (13)	.57 (7)	.75 (4)
Session 3	.64 (14)	.62 (13)	.51 (35)	.27 (22)	.50 (16)
Sum	.64 (39)	.66 (32)	.43 (77)	.38 (45)	.56 (36)
		Probability 1/2-1/2 onlv			
Session 1	.44 (9)	.56 (9)	.50 (16)	.33 (9)	.67 (6)
Session 3	.56 (9)	.57 (7)	.55 (22)	.21 (14)	.36 (11)
Sum	.50 (18)	.56 (16)	.53 (38)	.26 (23)	.47 (17)
		Probability 2/3-1/3 only			
Session 1	.70 (10)	.80 (5)	.38 (13)	.57 (7)	.50 (10)
Session 2	.83 (6)	.80 (5)	.15 (13)	.57 (7)	.75 (4)
Sum	.75 (16)	.80 (10)	.27 (26)	.57 (14)	.57 (14)
		Probability 9/10-1/10 only			
Session 3	.80 (5)	.67 (6)	.46 (13)	.38 (8)	.80 (5)

NOTE—A prediction is made only when expected utility prediction and actuarial prediction differ.

TABLE 4

Expected Utility — Minimax Comparison

.XX or .XXX = Correct Expected Utility Predictions/Total Predictions

(Y) = Total Predictions

Session	S01	S02	S03	S04	S05
Session 1	.76 (49)	.86 (56)	.76 (37)	.76 (51)	.98 (47)
Session 2	.94 (31)	.94 (34)	.79 (24)	.76 (29)	.94 (34)
Session 3	.87 (60)	.88 (64)	.44 (39)	.84 (50)	.91 (54)
Sum	.84 (140)	.88 (154)	.64 (100)	.79 (130)	.94 (135)
		Probability 1/2-1/2 only			
Session 1	.79 (24)	.89 (28)	.88 (16)	.91 (23)	1.00 (24)
Session 3	.84 (32)	.94 (35)	.38 (21)	.92 (25)	.83 (30)
Sum	.82 (56)	.92 (63)	.59 (37)	.92 (48)	.91 (54)
		Probability 2/3-1/3 only			
Session 1	.72 (25)	.82 (28)	.67 (21)	.64 (28)	.96 (23)
Session 2	.94 (31)	.94 (34)	.79 (24)	.76 (29)	.94 (34)
Sum	.84 (56)	.89 (62)	.73 (45)	.70 (57)	.95 (57)
		Probability 9/10-1/10 only			
Session 3	.89 (28)	.79 (29)	.50 (18)	.76 (25)	1.00 (24)

NOTE: A prediction is made only when expected utility prediction and minimax prediction differ.

TABLE 3 (cont.)

S06	S07	S08	S09	S10	Total
.84 (31)	.54 (28)	.74 (50)	.84 (57)	.87 (45)	.702 (305)
.54 (13)	.50 (14)	.86 (28)	.77 (31)	.95 (22)	.706 (143)
.70 (33)	.61 (31)	.53 (57)	.79 (53)	.90 (42)	.636 (316)
.73 (77)	.56 (73)	.67 (135)	.81 (141)	.90 (109)	.675 (764)
1.00 (14)	.33 (12)	.76 (25)	.97 (30)	.86 (22)	.717 (152)
.75 (20)	.82 (22)	.74 (35)	.86 (36)	.94 (31)	.710 (207)
.85 (34)	.65 (34)	.75 (60)	.91 (66)	.91 (53)	.713 (359)
.71 (17)	.69 (16)	.72 (25)	.70 (27)	.87 (23)	.686 (153)
.54 (13)	.50 (14)	.86 (28)	.77 (31)	.95 (22)	.706 (143)
.63 (30)	.60 (30)	.79 (53)	.74 (58)	.91 (45)	.696 (296)
.62 (13)	.11 (9)	.18 (22)	.65 (17)	.82 (11)	.495 (109)

TABLE 4 (cont.)

S06	S07	S08	S09	S10	Total
.78 (37)	.62 (39)	.50 (4)	.69 (13)	.53 (19)	.773 (352)
.88 (24)	.71 (24)	1.00 (3)	.67 (6)	.46 (13)	.833 (222)
.60 (40)	.74 (43)	1.00 (6)	.33 (18)	.31 (29)	.727 (403)
.73 (101)	.69 (106)	.85 (13)	.51 (37)	.41 (61)	.768 (977)
.84 (19)	.50 (20)	.00 (1)	.67 (3)	.86 (7)	.830 (165)
.44 (18)	.79 (19)	. (0)	.17 (6)	.22 (9)	.728 (195)
.65 (37)	.64 (39)	.00 (1)	.33 (9)	.50 (16)	.775 (360)
.72 (18)	.74 (19)	.67 (3)	.70 (10)	.33 (12)	.722 (187)
.88 (24)	.71 (24)	1.00 (3)	.67 (6)	.46 (13)	.833 (222)
.81 (42)	.72 (43)	.83 (6)	.69 (16)	.40 (25)	.782 (409)
.73 (22)	.71 (24)	1.00 (6)	.42 (12)	.35 (20)	.726 (208)

TABLE 5

Expected Utility—Actuarial Comparison

.XX or .XXX = Probability (Under the Null Hypothesis) of Getting at Least as Many Correct Expected Utility Predictions as Were Actually Obtained. (See Text.)

(Y) = Total Predictions

Session	S01	S02	S03	S04	S05
Session 1	.32 (19)	.21 (14)	.77 (29)	.77 (16)	.40 (16)
Session 2	.11 (6)	.19 (5)	1.00 (13)	.50 (7)	.31 (4)
Session 3	.21 (14)	.29 (13)	.50 (35)	.99 (22)	.60 (16)
Sum	.05 (39)	.06 (32)	.91 (77)	.96 (45)	.31 (36)
		Probability 1/2-1/2 only			
Session 1	.75 (9)	.50 (9)	.60 (16)	.91 (9)	.34 (6)
Session 3	.50 (9)	.50 (7)	.42 (22)	.99 (14)	.89 (11)
Sum	.59 (18)	.40 (16)	.44 (38)	.99 (23)	.69 (17)
		Probability 2/3-1/3 only			
Session 1	.17 (10)	.19 (5)	.87 (13)	.50 (7)	.62 (10)
Session 2	.11 (6)	.19 (5)	1.00 (13)	.50 (7)	.31 (4)
Sum	.04 (16)	.05 (10)	1.00 (26)	.40 (14)	.40 (14)
		Probability 9/10-1/10 only			
Session 3	.19 (5)	.34 (6)	.71 (13)	.86 (8)	.19 (5)

NOTE—A prediction is made only when expected utility prediction and actuarial prediction differ.

TABLE 6

Expected Utility — Minimax Comparison

.XX or .XXX = Probability (Under the Null Hypothesis) of Getting at Least as Many Correct Expected Utility Predictions as Were Actually Obtained. (See Text.)

(Y) = Total Predictions

Session	S01	S02	S03	S04	S05
Session 1	.00 (49)	.00 (56)	.00 (37)	.00 (51)	.00 (47)
Session 2	.00 (31)	.00 (34)	.00 (24)	.00 (29)	.00 (34)
Session 3	.00 (60)	.00 (64)	.83 (39)	.00 (50)	.00 (54)
Sum	.00 (140)	.00 (154)	.00 (100)	.00 (130)	.00 (135)
		Probability 1/2-1/2 only			
Session 1	.00 (24)	.00 (28)	.00 (16)	.00 (23)	.00 (24)
Session 3	.00 (32)	.00 (35)	.91 (21)	.00 (25)	.00 (30)
Sum	.00 (56)	.00 (63)	.16 (37)	.00 (48)	.00 (54)
		Probability 2/3-1/3 only			
Session 1	.02 (25)	.00 (28)	.09 (21)	.09 (28)	.00 (23)
Session 2	.00 (31)	.00 (34)	.00 (24)	.00 (29)	.00 (34)
Sum	.00 (56)	.00 (62)	.00 (45)	.00 (57)	.00 (57)
		Probability 9/10-1/10 only			
Session 3	.00 (28)	.00 (29)	.59 (18)	.01 (25)	.00 (24)

NOTE—A prediction is made only when expected utility prediction and minimax prediction differ.

TABLE 5 (cont.)

S06	S07	S08	S09	S10	Total
.00 (31)	.43 (28)	.00 (50)	.00 (57)	.00 (45)	.000 (305)
.50 (13)	.60 (14)	.00 (28)	.00 (31)	.00 (22)	.000 (143)
.02 (33)	.14 (31)	.40 (57)	.00 (53)	.00 (42)	.000 (316)
.00 (77)	.17 (73)	.00 (135)	.00 (141)	.00 (109)	.000 (764)
.00 (14)	.93 (12)	.01 (25)	.00 (30)	.00 (22)	.000 (152)
.02 (20)	.00 (22)	.00 (35)	.00 (36)	.00 (31)	.000 (207)
.00 (34)	.06 (34)	.00 (60)	.00 (66)	.00 (53)	.000 (359)
.07 (17)	.11 (16)	.02 (25)	.03 (27)	.00 (23)	.000 (153)
.50 (13)	.60 (14)	.00 (28)	.00 (31)	.00 (22)	.000 (143)
.10 (30)	.18 (30)	.00 (53)	.00 (58)	.00 (45)	.000 (296)
.29 (13)	1.00 (9)	1.00 (22)	.17 (17)	.03 (11)	.576 (109)

TABLE 6 (cont.)

S06	S07	S08	S09	S10	Total
.00 (37)	.10 (39)	.69 (4)	.13 (13)	.50 (19)	.000 (352)
.00 (24)	.03 (24)	.13 (3)	.34 (6)	.71 (13)	.000 (222)
.13 (40)	.00 (43)	.02 (6)	.95 (18)	.99 (29)	.000 (403)
.00 (101)	.00 (106)	.01 (13)	.50 (37)	.94 (61)	.000 (977)
.00 (19)	.59 (20)	1.00 (1)	.50 (3)	.06 (7)	.000 (165)
.76 (18)	.01 (19)	.00 (0)	.98 (6)	.98 (9)	.000 (195)
.05 (37)	.05 (39)	1.00 (1)	.91 (9)	.60 (16)	.000 (360)
.05 (18)	.03 (19)	.50 (3)	.17 (10)	.93 (12)	.000 (187)
.00 (24)	.03 (24)	.13 (3)	.34 (6)	.71 (13)	.000 (222)
.00 (42)	.00 (43)	.11 (6)	.11 (16)	.89 (25)	.000 (409)
.03 (22)	.03 (24)	.02 (6)	.81 (12)	.94 (20)	.000 (208)

Notes to Tables 3–6

Summaries of the prediction results are reported by subject, session, and probability. For each subject results for each session aggregated over all probabilities are reported in rows one, two, and three. The fourth row (labeled "Sum") shows the results aggregated over all probabilities and all sessions. Rows five to eleven contain results for the various probability types. The "Total" column gives results aggregated over all subjects.

a. Table 3 (4) reports the proportion of correct predictions by expected utility where expected utility and actuarial (minimax) indicate a prediction discrepancy. (If one of the techniques cannot make a prediction on a particular lottery pair, e.g., if expected utility difference equals zero, it is not counted as a discrepancy.)

b. Tables 5 and 6 show significance levels under the null hypothesis* for the proportions reported in Tables 3 and 4.

* The null hypothesis is that expected utility and actuarial (or minimax) are equal in predictive ability.

5.3 Results for Individual Subjects

In this section the utility assignments (see Figures 5.1–5.10)[1] and the prediction results will be discussed subject by subject. In addition, comments of the individual subjects will be interspersed where relevant, and any generalizations over subjects which seem justified will be made.

After the final choice session, each subject was interviewed briefly concerning his behavior during the choice sessions. In part, the purpose of these interviews was to note whether any verbal generalizations made by an individual are displayed by his estimated utility function. It was also of interest to obtain comments which could be useful in formulating future experiments.

Subjects were asked questions about the experiment, e.g., questions relating to subjective probability and to the use of simple decision rules. In addition, subjects were asked for information about themselves which, apart from the experimental process, might be relevant to choice-making under risk. These questions related to insurance coverage, educational background, and attitudes toward gambling.

From the results of the questionnaire, it appears that prediction accuracy was highly correlated with mathematical aptitude. Very little generalization could be made with respect to attitudes toward gambling, sources of financial support, or insurance. There did not seem to be any relationship, for example, between heavy insurance coverage and risk aversion nor any relationship between enjoyment of gambling and risk-loving or actuarial behavior.[2] However, the subjects who did not enjoy gambling generally display risk averse behavior.

Very few subjects made calculations, such as expected value, although expected value generally was acknowledged as a relevant consideration for the 1/2-1/2 lotteries. The subjects who made calculations tended to be "actuarial" subjects. It was evident that subjects were much more "at home" with 1/2-1/2 probabilities than with 2/3-1/3 or 9/10-1/10. This phenomenon may be explained partially by the fact that it is easier to verbalize the meaning of 1/2-1/2. I had the impression that there were subjective probability problems, but from conversation it was impossible to impute the direction or magnitude of the deviations from objective probabilities.

---

1. To facilitate visual comparison, the utility estimates have been normalized so that U($9.75) = 50 and U($0) = 0. Actual estimates are reported in Appendix A.

2. Although love of gambling may be frequently associated with risk-lovers, this generalization is probably not appropriate. Poker players, for example, who (it may be presumed) enjoy gambling, typically display actuarial behavior.

The remainder of this section is devoted to a discussion of the results, subject by subject. Subjects are numbered S01, S02, . . . , S10 only for identification purposes. The first four subjects are the graduate students; each student's department of study is enclosed in parentheses, e.g., S01 (history).

The utility function for S01 (history) looks very much like the second Monte Carlo theoretical function discussed in Section 3 (Figure 3.2). It also shows a marked resemblance to the estimated maximum likelihood utility function for the first theoretical function (Figure 3.1), except for the negative payoffs. S01's aversion to negative payoffs was evident in the interview session. The subject gave some indication that he made each choice without much regard to previous choices. He said he did not remember the lotteries he had drawn earlier, and there was some indication he thought he was to play only one lottery. Expected utility gave better prediction (at the 5% level) than actuarial for 39 discrepancies, and expected utility was better than minimax (at the 2% level) in all classifications, i.e., probabilities and sessions.

The utility assignments for S02 (philosophy) suggest a region of risk-loving behavior; moreover, during the interview period, he suggested that he frequently chose the risk-lottery when medium-size payoffs were involved. He expressed a desire to have more of his money at stake. At the first two sessions he drew

1/2	-$1.50	and	2/3	$9.75
1/2	$8.25		1/3	$2.75

respectively; consequently, according to his report, his behavior was even more risk loving on the third day. He expressed doubt after some of the sessions that his behavior had been consistent. It is interesting to note that this subject, the only risk-lover (age 24, married, no children) was enrolled in Blue Cross and had purchased $15,000 of life insurance. He was the only subject who suggested the experiment was boring. For this subject, expected utility gave better prediction (at the 6% level) than actuarial for 32 discrepancies. Expected utility exceeded minimax prediction in all classifications (at the 1% level).

The choices of S03 (religion) led to a peculiar looking utility function, and prediction for this subject was poor. No other subject had as low a percentage of correct predictions by expected utility (62%). Expected utility made fewer correct predictions than actuarial; however expected utility was better than minimax at the 1% level. At the beginning of the first session, S03 almost erred on one of the dominated lottery pairs. He claimed he was not at all concerned about consistency and changed his mind with respect to risk even within a session. At the final session, he announced he would be very risk averse, since he had just had an unfortunate experience in his German reading course. This was reflected in his predictions. On the first two

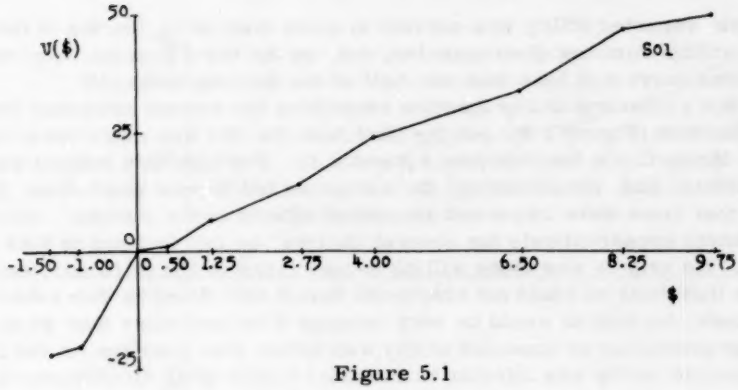


Figure 5.1

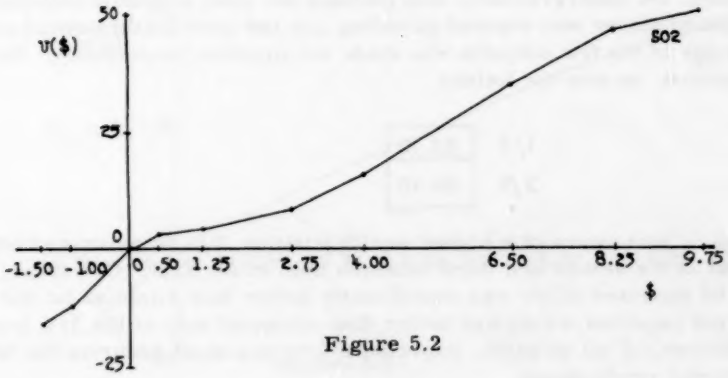


Figure 5.2

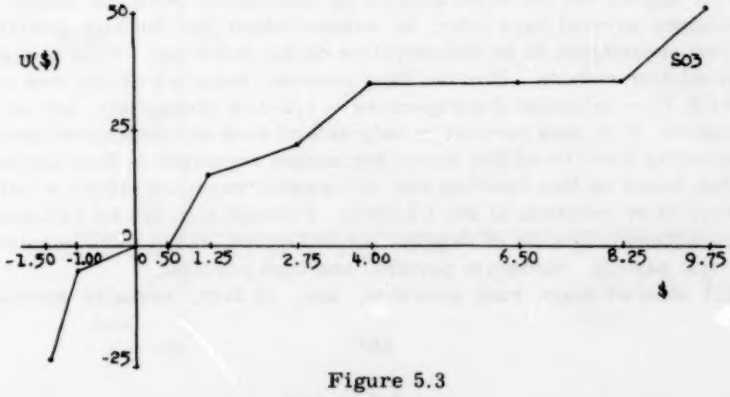


Figure 5.3

sessions, expected utility was correct in more than three-fourths of the expected utility-minimax discrepancies, yet, on the third session, expected utility was correct in less than one-half of the discrepancies.

S04's (history) utility function resembles the second estimated Monte Carlo function (Figure 3.2), yet the likelihood for S04 was much lower than for the Monte Carlo function (see Appendix A). Perhaps this subject was very inconsistent, and, consequently, the estimates led to poor prediction. He suggested that there were important sequential effects in his choices. After he had behaved conservatively for several choices, he felt inclined to take a chance. He said he was more willing to take risks on the third day, since he knew at that point he could not lose more than $.25. When he was asked to participate, he said he would be very unhappy if he lost more than $4 or $5. Although prediction by expected utility was better than minimax (at the 1% level), expected utility was inferior to actuarial in 62% of 45 discrepancies. This was the poorest performance of all subjects in the expected utility-actuarial comparison.

S05, the undergraduate, was perhaps the most actuarial subject. He was a poker player who enjoyed gambling and had good mathematical aptitude. He was one of the few subjects who made calculations consistently. On the first session, he won the lottery

1/3	$2.75
2/3	$6.50

Although he was aware of a higher wealth position, his behavior was not more actuarial in the second and third sessions than in the first. Of course, prediction by expected utility was significantly better than minimax (at the 1% level), but expected utility was better than actuarial only at the 31% level. Nevertheless, of all subjects, expected utility was most accurate for S05 (89% correct predictions).

The remaining five subjects were from the Air Force Language School. It will be noticed that they were more risk averse than the graduate students. S06 did not appear for the third session as scheduled. After he completed the third sessions several days later, he acknowledged that the high probability of loss had caused him to be conservative on the third day. This is displayed in the prediction results. For the first session, expected utility was correct in 84% of E.U. — minimax discrepancies — 1/2-1/2 probability, but on the third session, E.U. was correct in only 44% of such discrepancies (see Table 4). The utility function of S06 shows horizontal segments at four levels. Prediction based on this function was successful; expected utility is better than actuarial or minimax at the 1% level. Perhaps this can be rationalized by a compartmentalization of payoffs. In his world, there were negative payoffs, trivial payoffs, moderate payoffs, and high payoffs.

S07 showed mark risk aversion, and, in fact, verbally stated

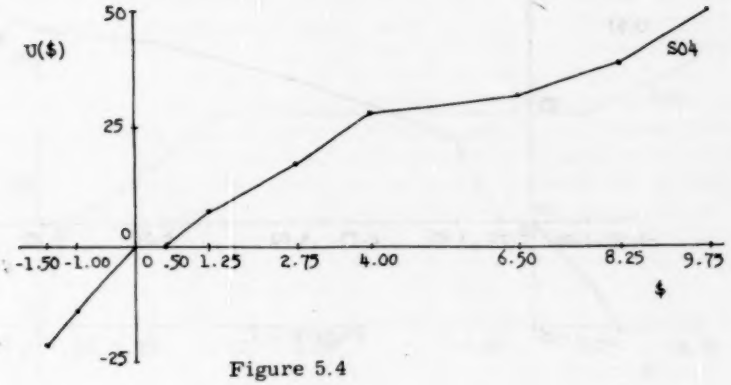


Figure 5.4

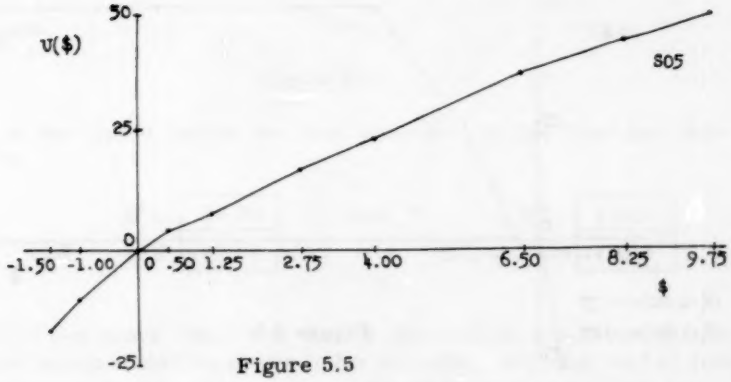


Figure 5.5

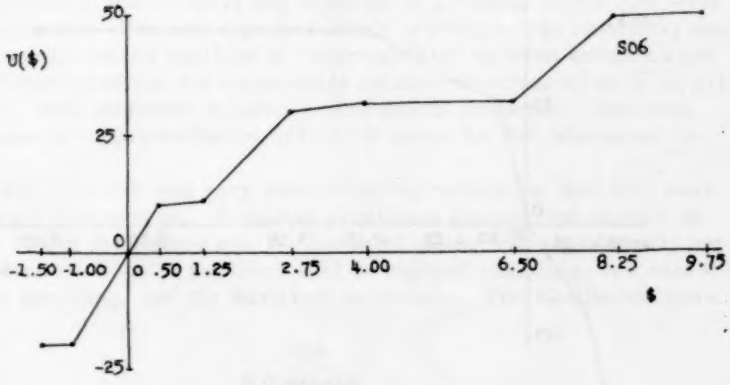


Figure 5.6

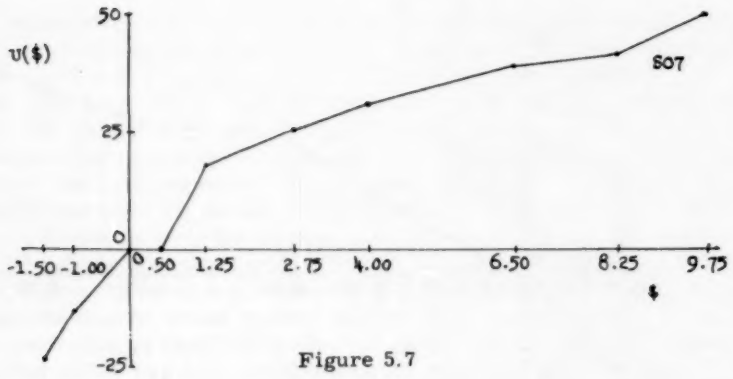


Figure 5.7

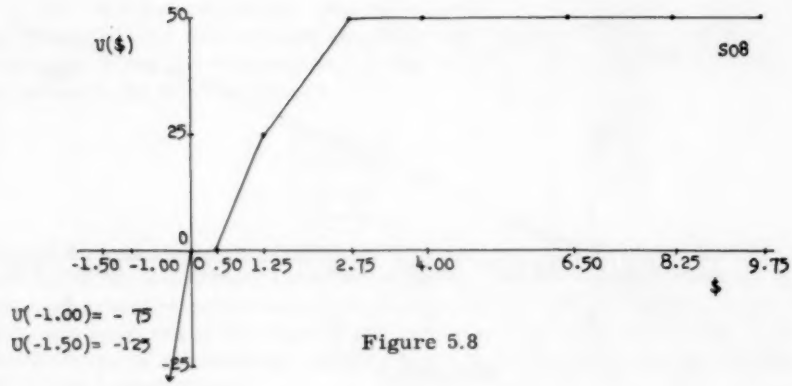


Figure 5.8

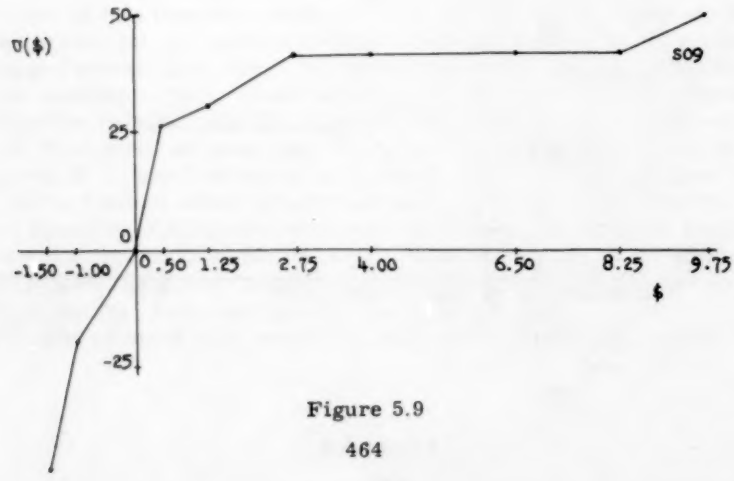


Figure 5.9

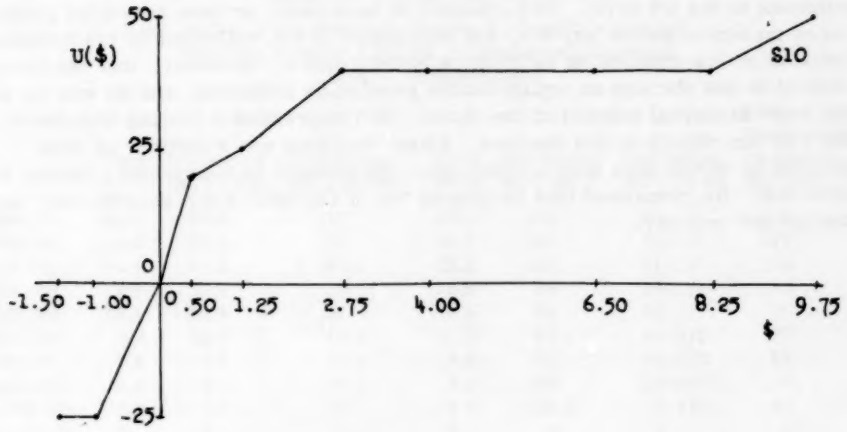


Figure 5.10

something to that effect before the first session. In the first two sessions he won

1/2	$6.50	and	2/3	$1.25
1/2	-$1.00		1/3	$9.75

He claimed he took more chances on the last day; but the prediction results do not reflect a significant departure in his behavior. Although he had some difficulty understanding the game and said his mathematical aptitude was "poor," his prediction results were satisfactory. Expected utility was superior to minimax at the 1% level and superior to actuarial at the 17% level on 73 discrepancies. Perhaps expected utility prediction was relatively successful because his utility function is "intermediate" between actuarial and minimax. Expected utility did very poorly relative to actuarial on 9/10-1/10 lottery pairs. This suggests subjective probability problems. Expected utility also gave poor prediction on 9/10-1/10 pairs for S08 (discussed below).

S08, S09, and S10 had very similar utility estimates, and they were overwhelmingly risk averse. Actuarial prediction was correct in only ap proximately 50% of the lottery pairs. Expected utility was significantly better than actuarial. None of the three said he enjoyed gambling—two said they did not enjoy gambling, and the third had no opinion. The likelihoods were

high for all three, but minimax led to more correct predictions than expected utility for S10.[1] For S08, on the other hand, expected utility was better than minimax at the 1% level. S08 claimed to have more or less actuarial preferences on non-negative payoffs, but this claim is not reflected by his estimated function or his choices on estimation lottery pairs. However, this tendency is evident in his choices on equiprobable prediction lotteries, and he was by far the most actuarial subject of the three. S08 expressed a feeling that there were order effects to his choices. After "building up" a number of safe choices he would then take a risky one. He thought he took more chances the first day. He remarked that he played "as if the odds were against me," and that he felt unlucky.

---

1. Likelihood equaled one for S09 and S10. This result means utility assignments were found which suggest no inconsistent or indifferent choices.

Appendix A: Utility Estimates

Monte Carlo Study (See Section 3.3)

Key: MC1 = First Function (Figure 3.1); MC2 = Second Function (Figure 3.2); ML = maximum likelihood; M-S = Manne-Suppes

	True MC1	Estimates MC1 (ML)	Estimates MC1 (M-S)	True MC2	Estimates MC2 (ML)	Estimates MC2 (M-S)	Non-Restricted Estimates MC2 (ML)
U($9.75)	20.2	67.6	50	25.0	154	50	168
U($8.25)	18.6	63.8	50	23.8	133	41-2/3	147
U($6.50)	16.4	51.2	37.5	22.2	133	41-2/3	154
U($4.00)	12.6	36.6	25	18.6	98	29-1/6	112
U($2.75)	10.4	32.8	25	14.6	84	25	98
U($1.25)	7.2	20.2	12.5	11.0	69.1	20-5/6	77
U($0.50)	5.4	17.3	12.5	9.4	56	16-2/3	63
U($0.00)	4.2	15.5	12.5	9.0	56	16-2/3	70
U(-$1.00)	1.4	5.6	0	3.0	21.9	4-1/6	28
U(-$1.50)	0	0	0	0	0	0	0
Value of Objective Function		24.0			31.6		32.6

Experimental Estimates (See Section 5)

	S01	S02	S03	S04	S05	S06	S07	S08	S09	S10
U($9.75)	38.6	165.3	16.7	19.0	52.7	55.85	34.4	49.0	175.0	105.0
U($8.25)	37.1	156.3	13.2	16.1	48.6	55.3	30.3	49.0	161.0	91.0
U($6.50)	30.4	127.5	13.2	14.3	43.0	40.95	29.2	49.0	161.0	91.0
U($4.00)	22.2	79.6	13.2	13.2	32.2	40.95	25.4	49.0	161.0	91.0
U($2.75)	19.6	60.7	10.3	10.3	27.1	39.5	22.7	49.0	161.0	91.0
U($1.25)	15.5	51.7	8.8	7.6	19.6	24.25	19.3	42.0	140.0	70.0
U($0.50)	12.3	47.9	5.3	5.6	16.8	23.7	10.8	35.0	133.0	63.0
U($0.00)	11.7	39.8	5.3	5.6	13.6	15.8	10.8	35.0	84.0	35.0
U(-$1.00)	.9	9.0	4.4	1.8	5.1	0	4.7	14.0	49.0	0
U(-$1.50)	0	0	0	0	0	0	0	0	0	0
Value of Objective Function	22.4	28.8	14.3	15.6	24.6	26.7	24.1	31.9	34.6	34.6

Appendix B: Notes on Experiment

1. Solicitation of Subjects

The Air Force subjects were solicited in the following way. Permission was obtained from the commanding officer of the Detachment[1] and, at his suggestion, a notice was posted on their bulletin board. The notice read as follows:

> MEN NEEDED FOR DECISION-MAKING PROJECT
>
> Requires approximately one hour Monday, Wednesday and Friday OR Tuesday, Thursday and Saturday. Next week (June 25–30). Profitable. Sign below or call F. T. Dolbear, ST 7-3131, ext. 507, or HU8-7337.

Within several hours 14 men signed the notice, and one telephoned. One subject was chosen from each of the three dormitories, and one of these three recommended a friend who had signed up. (As far as I know, these two were the only subjects who knew another participant.) The fifth subject did not live in the dormitories.

To induce a subject to participate, it was thought necessary to give him some information concerning the payoff matrix. He was told he could win as much as $29.25 or lose as much as $4.50 for approximately three hours' work. He was told also that his chances of losing money were less than 2 in 100, and that expected winnings were between $7 and $12. When given the above information, none of the Air Force men refused to participate.

Four Yale graduate students were obtained through contacts in various departments in the Division of the Humanities. Several additional graduate students declined to participate, typically because of "other commitments" and "pressing academic responsibilities." The fifth Yale student, an undergraduate senior, was obtained through the Yale Financial Aids Office.[2] I have little reason to believe that I received subjects from the two groups under consideration (i.e., Air Force Language School and graduate students) who were eager (relative to the groups as a whole) to play a gambling game.

2. Experimental Procedure

Each subject was tested individually. At the beginning of each session, a subject was given a stack of approximately 50 cards; each card contained two lotteries. After indicating a choice by placing a check below the chosen lottery, the subject dropped the card in a small box and continued to the next

---

1. The commanding officer was told that the subjects would not lose money; but, of course, the subjects themselves were not informed of this decision.

2. It was my original intention to obtain five graduate students. The Financial Aids Office billed its subject as a law student, but he turned out later to be an undergraduate who had law school aspirations.

card. He was not permitted to look at previous choices. After the first 50 cards were completed, the subject was given a second and final stack of approximately 50 cards. After these were marked and dropped into the box, all the cards were mixed and the subject drew one of the cards at random from the box. The chosen lottery on that card was recorded, and the subject was given a copy of the lottery which he had drawn for the session. However, no lottery was actually played until the conclusion of the third session, when the subject played three lotteries (one from each session), and was paid the total amount he had won.

A lottery was played by drawing a card from a box containing cards marked "TOP" or "BOTTOM". The proportion of each type of card represented the probability of the top payoff and of the bottom payoff of that lottery. If a card marked "TOP" was drawn, the top payoff was received; but if a card marked "BOTTOM" was drawn, the bottom payoff was received. This probability mechanism appeared to cause no difficulty. A technique such as ZEJ-ZOJ (syllables which have practically no association value, and which were used by Davidson, Suppes, and Siegel in their experiments) was contemplated to minimize subjective probability difficulties. However, it was decided there would be little if any distortion with TOP-BOTTOM, and this technique had a mnemonic advantage.

Summary: In recent years intensive efforts have been made by certain Soviet economists and mathematicians to find ways of improving national-economic planning methods through the use of input-output analysis and linear programming. In this essay an attempt is made to outline the work of the major contributors to these efforts, and to discuss the impact of this work on Soviet economic theory and practice.

For reasons discussed in the essay mathematical techniques present a sharp challenge to the Marxian framework within which Soviet economic science has long existed. A substantial portion of the essay is therefore devoted to an evaluation of the numerous criticisms leveled by the Marxian orthodoxy at the proponents of mathematical techniques. In conclusion an attempt is made to predict the future of mathematical techniques in the USSR in light of the problems involved in planning an economy which can be argued to be reaching the end of a "development stage" and entering a more mature and complex stage of existence.

This essay was submitted in May 1963 in partial fulfillment of the requirements of the Honors major in economics for the Bachelor of Arts degree in Yale College. It was awarded the Charles Heber Dickerman Memorial Prize for the best departmental essay written by a senior majoring in economics. The author is currently a graduate student at the University of California at Berkeley.

ON THE APPLICATION OF MATHEMATICS IN SOVIET ECONOMICS

Alan Parker

1. Introduction

When Karl Marx set out to demonstrate the immorality of the economic institutions prevailing in nineteenth-century industrial England, he found the labor theory of value both a useful instrument to this end and a doctrine in large measure acceptable to the thinkers of the time. While one profitably may question the relevance of many of the traditional Marxian categories to the life of the modern Soviet state, it is abundantly clear that Marx's use of the labor theory of value has profoundly influenced the course and content of Soviet thought on economics.

Within the last few years, however, the traditional framework has been challenged. A pressing need for more satisfactory planning methods and the increasing relevance of advanced mathematical technique to the planning process have worked together to bring about a growing awareness of the inadequacy of the traditional concepts.

The purpose of this paper is an examination of the nature of the work being done in the Soviet Union in the field of mathematical economics, the debate surrounding much of this work, and the implications of these developments for the future of economic theory and policy in the Soviet Union.

Basically, the labor theory of value is employed in the Soviet context to dispel "subjective" concepts of value and to "objectify" the value content of a good as equal to the value of total expenditures of "socially necessary labor" involved in its production. Thus the value of a good can be objectively ascertained; it is not dependent upon the quantity of the good available; and it can be expressed in units of homogeneous labor expenditure. For example, the value of a ton of coal is proportionate to the quantity of social labor expended in its production; the value of a capital good is proportionate to the quantity of social labor expended in its manufacture, and, over its lifetime, the capital good surrenders the total of this "embodied labor" to the goods which it helps create.

In the Soviet Union, goods within the socialist sector, that is, goods which are exchanged only among state-owned enterprises, are given a ruble price which corresponds to the average cost of producing the good, and which is felt to be approximately proportional to the good's content of social labor.

If labor is the source of all value and a capital good is valued at its content of social labor, there can be no charge attached to the use of capital; hence there is no explicit interest rate or capital charge to the user on the bulk of investment funds in the USSR.

The difficulty of long-term or investment planning without a capital charge has been apparent for a long time to both the Soviets and to foreign

observers. Engineers in the USSR charged with the task of choosing between technological variants of a given investment project have consistently favored solutions involving technologically more advanced techniques, and have even developed pseudocapital-charge schemes to avoid the absurdly capital-intensive solutions possible when capital is considered a free good.[1]

Current, or short-term, planning is divorced to a large extent, at least theoretically, from the problem of "value."[2] The plan by which the economy is to operate is drawn up insofar as it proved possible in physical terms. Coefficients or "norms" relating physical inputs to output in tons or thousands of liters are employed to construct "balances" for each planned commodity. The planning agency attempts to plan production in such a way as to make total output of the economy equal to total inputs plus final demand for the plan period.

The method used by the Soviets to construct such a plan is called the method of balances. Soviet enterprises submit estimates of next-period output together with estimates (based on the aforementioned norms) of the inputs required for the projected output. The planning agency then balances the input requirements and the promised outputs. Generally, this balance is not obtained on the first attempt, since demanded inputs usually exceed the projected supply. It is the planning agency's task to bring demand into line with supply by decreeing changes in the quantities which enterprises will supply and demand. Producers of goods for which there exists a deficit in the plan are asked to produce more of the good. Their suppliers are in turn requested to produce more, and so on, until the relevant dependencies within the plan are traced out and total demand is matched with supply.

In actual practice, however, the chains of dependencies are rarely followed past the level of determining increases in inputs into the original deficit good. This means that the balances struck will be less accurate or realistic the greater the extent of interdependencies or "feedback" effects in the system.

In Soviet terminology second, third, and higher level dependencies are the basis of "indirect" costs of the social product.[3] Work is currently under way to better calculate these indirect costs, that is, to improve plan consist-

---

1. Gregory Grossman, "Scarce Capital and Soviet Doctrine," Quarterly Journal of Economics, Vol. LXVII, No. 3 (August 1953), pp. 311–43.

2. For a short discussion of Soviet planning practice, see Herbert S. Levine's "Input-output Analysis and Soviet Planning," American Economic Review, Proceedings (May 1962).

3. N. I. Kovalev, "The Problems of Introducing Mathematics and Electronic Computers in Planning," Problems of Economics, Vol. V, No. 4 (translation from Voprosy ekonomiki, No. 12, 1962).

ency through the construction of inter-branch input-output (I-O) matrices. At present, the Soviets plan to calculate global I-O coefficients as soon as the planners acquire sufficient information about production processes.[1]

In 1959, the Central Statistical Bureau constructed a table indicating production and distribution in the Soviet economy. From this table an 87 × 87 item matrix in value terms and a 157 × 157 physical balance were compiled.[2] In 1961, the Computer Center of the State Economic Research Council and the Economic Research and Planning Institute of the Soviet Academy of Sciences were working together to compile a 350 × 350 inter-branch production matrix.[3] The ultimate goal of this project, according to Kovalev, is to make such matrices one of the regular tools of the planning apparatus.

Thus, the Soviets are directing a great deal of effort toward the adaptation of their method of balances to modern techniques, namely I-O analysis. Coupled with modern computer technology, I-O analysis could facilitate the manipulation of large numbers of balances, and envisioned in the future is the creation of a vast information-gathering and -processing network manned by ranks of electronic computers in Moscow and other central points.[4] At any time the planners could ascertain that level at which a process would have to operate in order to fulfill a given schedule of final demands. In other words, the planning agency could work out a completely consistent plan, one in which inputs and outputs would be accurately balanced.

The problem of planning, however, is not solved simply by the construction of a consistent plan, even granting that the economy could be made to perform in accordance with the plan. This fact is brought to light by further investigation of current Soviet practice. The quest for global I-O coefficients noted above is the result of a desire on the part of mathematical economists to construct plan "variants." Given information relating to the stock of resources, capital goods, and given the relevant process coefficients, it becomes possible to plug in alternative systems of gross and final demand to create various achievable plans. At present, the Soviets are studying the possibility of doing just this.[5] But, the enumeration of plan variants does not solve the planning problem. This point can be easily demonstrated. Put simply, systems involving capacity limitations or limited supplies of factors are

---

1. Ibid.

2. Ibid.

3. Ibid.

4. See I. S. Bruk in V. S. Nemchinov, ed., Obshchie voprosy primenenie matematiki v ekonomike i planirovanii (Moscow: Isdatel'stvo Akademii Nauk, 1961).

5. J. M. Montias, "Planning with Material Balances," American Economic Review, Vol. XLIX (December 1959).

not solved by I-O analysis; the area of application of these techniques is confined to the analysis of general economic equilibrium.[1]

Let it be supposed, for example, that two plan variants, each adequately expressed as closed I-O systems generate the same final bill of goods. The system under either plan is not producing at full capacity, i.e., not efficiently produced. In traditional terms, the system is not producing on the production possibility frontier, but somewhere inside it. Both combinations of processes —both plans—may be consistent, but an increase in output is possible through a reallocation of resources and means of production.

It is difficult to assume, as J. M. Montias has pointed out, that the planning agency attaches zero marginal utility to output above that of the selected plan variant.[2] Therefore "efficiency" in the sense used above plays a role in the planners' hierarchy of values.

How do Soviet planners forge the labor theory of value and the method of balances with its new expression in I-O methodology into a logical and useful theory of resource allocation? What kind of guideposts do planners follow in order to move toward a situation in which factors of production are employed where their marginal contribution to total output could be no greater under existing conditions?

The answer is that no synthesis of the labor theory of value and allocation theory is made; planners are without all but the crudest guides to efficient allocation. Soviet economic theory is without tools to attack the problem of economic efficiency. Prices which fail to reflect relative scarcities in a universe of scarce goods serve only as a bookkeeping convenience in a system which chooses to ignore on a theoretical level the whole problem of efficient resource allocation.

2. The Soviets and Mathematical Methods

It is precisely in connection with the problem of allocative efficiency that the most spectacular clash occurs between Soviet ideology and the requirements of rational, effective planning. In recent years, major breakthroughs have occurred in the formulation and solution of a class of mathematical problems which involves the minimization or maximization of sets of variables subject to various constraints expressed as linear inequalities, while the rapid development of computer technology has facilitated the actual manipulation and solution of large systems of equations and inequalities. The application of these techniques in economics is called linear programming.

---

1. Robert Dorfman, "Mathematical, or 'Linear,' Programming: A Non-Mathematical Exposition," American Economic Review, Vol. XLIII (December 1953).

2. In lecture, Spring 1963.

In the West the similarities between the theoretical foundations of linear programming and the traditional marginal calculus have been noted and exploited. Also, Soviet efforts in this field have paid off in the form of a profound and competent comprehension of this class of problem. The Soviets have made impressive contributions to the theory of linear programming; indeed, it can be argued that L. V. Kantorovich, professor of mathematics at Leningrad State University and member of the Soviet Academy of Sciences, anticipated Western work in the field by several years and is the founder of linear programming.

Soviet contributions to the mathematics of linear programming notwithstanding, the path from the formulation to the application of these techniques, and in general to investing them with real economic content, has been steep and treacherous for those who would travel it. The challenge to the traditional doctrine has been felt despite protestations of fidelity by the proponents of the new analysis.

3. Novozhilov and Kantorovich—Their Systems of Analysis

V. V. Novozhilov's substantial contribution to Soviet economic thought is his concept of the "inversely related costs" of production.[1] He argues that, given the existing system of prices (ruble prices reflecting average outlays of social labor), the use of a process which secures the smallest expenditure on a given product may involve increases in the costs of other products (due to the denial to their production of certain inputs necessary to the chosen process of production of the given product) which offset or exceed the saving gained by employing the advantageous process. From this point, he reasons that such costs must be determined and added to the cost of all inputs. Only then will producers' individual cost minima correspond to the cost minimum in the economy as a whole.

The student of Western economic theory has already noted that Novozhilov's formulation bears a great resemblance to the logic of opportunity costs and scarcity pricing. Novozhilov in fact is arguing for a scarcity price system of a sort, but he preserves a measure of ideological respectability by making socially necessary labor the numéraire of his system—the unit in terms of which all commodities are expressible—and the repository of value.

Given a set of final demands, the existing stock of productive resources, plant and equipment, the planned bill of investment for the relative period—all potentially expressible in units of social labor—the task set is to minimize total expenditures of social labor. In the process of solving the system for minimum expenditures, equilibrium prices for intermediate goods, scarcity

---

1. Novozhilov's basic formulation is found in Papers of the Leningrad Industrial Institute (1939) and Papers of the Leningrad Polytechnic Institute (1946). These are combined and translated as "On Choosing between Investment Projects" in International Economic Papers, Vol. II (1956).

rents, and an interest rate on capital are all generated. These prices consist of the sum of current expenditures of labor, expenditures of "embodied labor," and the "inversely related costs" of production.

The inversely related cost associated with a unit of input is equal to the difference between the saving of labor (in hours per unit output) effected in its least advantageous actual application and the saving of labor which would result if the resource were employed to no advantage in the economy.

The student of Western economics again encounters familiar logic. If there are more opportunities for the application of a resource than can be met with existing supplies, it makes sense to employ the resource first where it effects the greatest saving of social labor (where its product is greatest). The least advantageous actual application of the resource corresponds to the marginal application of the resource. If, for any unit application of the resource, the cost of current labor expenditures per unit output plus a charge reflecting the difference between the saving effected by the marginal application of the input and the worst possible application (marginal product equal to zero, or no saving effected) is greater than for another application, then that application does not correspond to an efficient allocation of that resource; per unit physical product of the resource is less than per unit physical product at the national margin.

Simile modo, the logic is applied to capital to yield an interest charge, and to existing stocks of plant and equipment to yield rental charges.

Optimal allocation is therefore achieved if individual producers minimize costs, taking costs to include the "inversely related costs" of production. Cost minimization can be seen to be the same as profit maximization; implementation of the Novozhilov model might therefore imply active profit maximization by producers. This same maximization, however, might be carried out on paper in the planning agency and, accordingly, physical allocations made.

Western theoreticians might be content with Novozhilov's formulation except for the fact that, possibly, Soviet economists are too content with it. This state of affairs arises from the fact that Novozhilov chooses hours of labor as his repository of value while hours of value are entering the production process. Minimizing labor inputs (maximizing leisure) does not generate a scarcity price for labor or a transformation curve between labor and other factor inputs.

Thus Novozhilov has constructed a system which yields scarcity prices for all factor inputs but labor and one which relies on the feasibility and desirability of (1) leisure maximization, and (2) fixed bills of investment and final demand.

By ingenious use of the traditional categories, Novozhilov is able to create a model which, on its face, resolves the contradictions between scarcity pricing and the labor theory of value. The costs, however, of the synthesis are great. In the first place, it is difficult to imagine that the goal of leisure maximization corresponds in any meaningful way to Soviet economic reality. On the contrary, the obvious and chief goal of the planners is output

maximization. If labor is believed to be the source of all value, then planners will maximize value added by maximizing labor inputs within limits imposed by the capacity and willingness of the labor force to expend social labor.

Another major shortcoming of the model, noted above, is its failure, due to the labor-power numéraire, to generate a transformation schedule between labor and other factor inputs. If limited non-labor resources are reallocated in such a way as to maximize reductions in labor inputs, gains will be made, but the new allocation plan is not necessarily optimal. This can be understood when it is realized that a reallocation of the labor supply following the reallocation of the (now fixed) capital supply might yield further savings. These would be in the form of unused plant capacity. Further alternate reallocations of capital and labor would presumably guarantee an approach to an optimal (least cost) production pattern of the fixed bill of demanded goods.

From the foregoing, it is seen that labor allocation according to the Novozhilov scheme would be arbitrary and therefore probably non-optimal. This attempt to define a valid allocative algorithm must therefore be judged a failure, but it represents, nonetheless, a significant step toward an understanding of scarcity pricing and the application of mathematical models in economics.

It is left to the mathematician mentioned earlier, L. V. Kantorovich, to fully apply the logic of linear programming to economics under Soviet conditions and deal the labor theory of value a mortal blow. Kantorovich's renown in economic circles in the USSR came relatively recently with the publication of his book, Ekonomicheskii raschet nailuchshego ispol'zovania resursov.[1] In it, he has outlined linear programming and his conception of its possible application to the planned Soviet economy. Central to his presentation are his "objectively conditioned estimates" (o.c.e.'s). These o.c.e.'s are shadow prices; they represent equilibrium marginal rates of transformation between goods, both final and intermediate. They are generated out of a system in which constraints reflecting supplies of labor, capital, and resources are expressed as systems of inequalities; output is maximized, subject to the condition that products are produced in fixed quantitative (planned) proportion to one another.

The nature of o.c.e.'s may be better understood by considering the graphical representation of a linear programming problem in which there are two commodities and three processes, with different relative efficiencies, capable of producing both commodities. (See Figure 1.) There is a limit to the total output of each process. Process I produces M relatively more efficiently than Process II (using total possible M / total possible N as a standard

---

1. L. V. Kantorovich, Ekonomicheskii raschet nailuchshego ispol'zovania resursov (The Economic Calculation of Optimal Resource Employment), Moscow, 1959.

of comparison), which still produces M relatively more efficiently than does Process III. The assortment plan calls for M to be produced in the proportion m/n to N.

The significance of line abcd is similar to that of a production possibility curve for M and N. Any point on abcd, therefore, represents optimal performance in the three-process, two-output economy depicted; i.e., for any point on abcd, no additional output of either good can be produced without sacrificing production of the other output. The only additional task is to fix the proportion in which outputs are to be produced in the "optimal" plan. Any point on the "assortment ray" is a point representing output in the stipulated proportion, m/n. Therefore, the intersection between the assortment ray and abcd indicates that m' and n' are the optimal outputs of M and N.

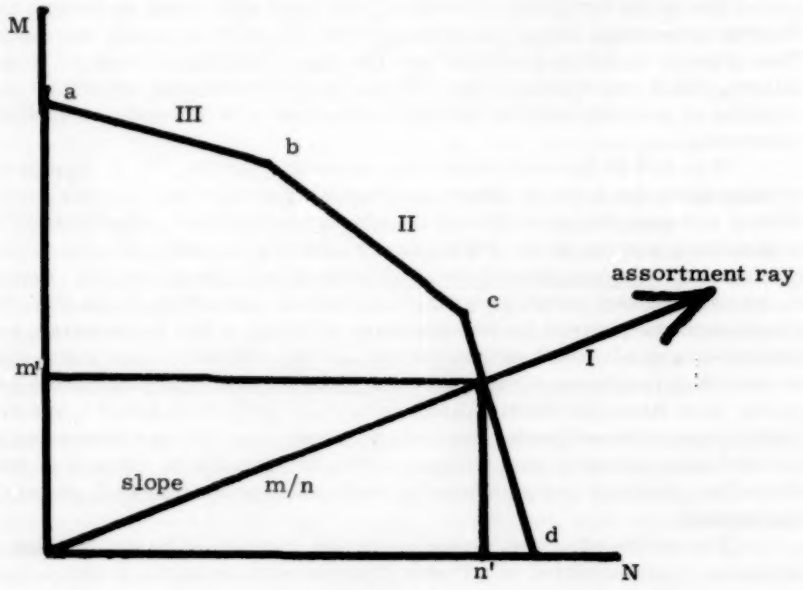


Figure 1

Thus, recalling equilibrium conditions under the assumptions of the marginal calculus,

$$MRT_{mn} = \frac{P_m}{P_n} = MRS_{mn}$$

it can be seen that the slope of the facet of the production envelope penetrated by an assortment ray has significance for the equilibrium "terms of trade" between M and N; namely,

$$\text{slope of penetrated facet} = \frac{\text{equilibrium price}_m}{\text{equilibrium price}_n} = \text{o.c.e.}_{mn}$$

Process I in the foregoing analysis produces both M and N. The other processes are confined to the production of M. Since the optimal solution demands that this be the case, the slope of cd reflects in a real sense the marginal rate of transformation of M into N.

This marginal rate of transformation, generated out of and dictated by the existing stock of productive resources, is an index which planners may use to tell how much M must be given up to secure an additional unit of N when the assortment plan is observed. Planners then have a guide to the relative "value" of M and N in the economy. Taking these "objectively conditioned estimates" of the relative value of the goods as prices, planners can reduce the problem of optimal resource allocation to that of choosing the least-cost process(es) from available alternatives. Once a set of o.c.e.'s is established, it may be used to determine whether a given plan represents an optimal allocation of resources. In a restricted sense, supply will not equal demand at going prices, i.e., at the "going" o.c.e.'s, if the plan is not optimal. O.c.e.'s can also be used to determine a new optimum if small changes are made in the problem, that is, if the assortment ray does not penetrate a different facet of the production envelope under the changed conditions.

Another possibility suggested by the Kantorovich model is that industrial prices could actually be set throughout the economy in such a way as to correspond to o.c.e.'s. Individual producers could then be instructed to maximize profits (minimize costs). (See Figure 1.) If the prevailing "price" ratio of M and N is slope cd, those in charge of Process III will find it much more "profitable" to produce M than to produce N. The assortment plan, therefore, is reinforced.

Under present conditions, a number of conflicting sets of cues and incentives compete for the attention of the Soviet enterprise director. Among the strongest of these is the incentive not to produce according to the assortment plan laid out by the planners.[1] Substitution of o.c.e.'s for the prices presently used would improve this situation and allow the planning center to relax to some extent the rigid controls necessitated by conflicting cues to producers.

Implementation of either the Novozhilov or the Kantorovich scheme would entail a complete revamping of the price structure of the USSR or the creation of a second system of prices to coexist with the present one. Also, a rejection of the whole Marxian framework obviously would be required. It is interesting that, in their work, both men often refer to actual practice in

---

1. See Joseph Berliner, Factory and Manager in the USSR (Cambridge, Mass., 1957).

the Soviet Union by which occasionally prices are set to achieve more rational allocation of resources.[1] Both stress the "all or nothing" character of their systems, emphasizing that the significance of the analysis is lost if it is applied only in parts, sectors, or areas of the national economy.

4. The Attack on the Positions of Novozhilov and Kantorovich

Despite the logic and reason of their positions, both men have encountered stubborn and sometimes bitter resistance. In recent years most of this criticism has been directed at Kantorovich. As Novozhilov correctly pointed out, theoretical and doctrinal opposition to linear programming methods (he was referring to opposition to Kantorovich) is of two main kinds.[2] First, there is the objection to the implications of the analysis for price and value theory; second, there are doubts about the applicability of linear programming to the problem of planning over time, i.e., dynamic planning.

Under the first heading are to be found criticisms of many kinds and on many levels. On the most simple level are arguments which attempt to show that the proponents of mathematical analysis hold a heretical view. A. Boiarskii, for example, accuses Kantorovich of "subjecting all the factors of production to . . . estimations . . . most reminiscent of marginalism."[3] Page after sterile page in the economic journals is dedicated to discussion of the relation of the "law of value" to prices in general and "objectively conditioned estimates" in particular, the main objection being that, since they do not measure inputs of social labor in any satisfactory sense, they cannot be tolerated as a general equivalent of exchange. Among the writers propounding more extreme positions, this discussion is usually ended with a suggestion that the mathematicians confine their work to their own field or become more familiar with the true complexities and "qualitative aspects" of economics.

As noted above, implementation of o.c.e.'s, or inversely related costs, make profitability calculation the relevant guide to national-economic efficiency. Criticisms of the profit criterion, while similar to the purely doctrinal objections already cited, evoke differing replies on the part of Novozhilov and Kantorovich. The former, on the firmer ground permitted by his recognition of the labor theory of value, argues that profitability is a correct criterion because it represents a reformulation of the basic premise of the whole argument, i.e., savings of social labor to the economy. The latter insists that profitability is not the question at all, that o.c.e.'s are merely

---

1. See L. V. Kantorovich in Kronrod, ed., Zakon stoimosti i ego ispol'zovanie v varodnom khoziaistve SSSR (Moscow: Gospolizdat, 1959), p. 289.

2. See V. V. Novozhilov in V. S. Nemchinov, ed., op. cit., p. 271.

3. A. Boiarskii, "O primenenii matematiki v ekonomike," Voprosy ekonomiki, No. 2 (1961).

a new and useful kind of economic "index" involved in the process of determining economic optima, that o. c. e.'s emerge from the deliberate and conscious planning possible only under socialism and are not the independent data of capitalist economic anarchy.

The application of o. c. e.'s as actual industrial prices is not discussed in Kantorovich's work. It is proposed, rather, by opponents who, to their horror, envision a kind of "market socialism in reverse"[1] made possible by the employment of o. c. e.'s as industrial prices. Kantorovich's main concern lies with the improvement of planning and not, as his critics might argue, with the subversion of Marxian value theory. Throughout his discussion, Kantorovich continually assures the reader that his intention is not to invest his o. c. e.'s with economic content, that they are only a tool to secure optimal resource allocation. The attempt, however, to reconcile the system with the labor theory of value damns him completely in the eyes of the orthodox. He argues that the optimal plan is the socially necessary one. He then attempts, albeit half-heartedly and preeminently unsuccessfully, to argue that a product contains socially necessary labor in proportion to the magnitude of its o. c, e. The result of the final proposition is to reveal that, indeed, Kantorovich would prefer to see o. c. e.'s become a surrogate for value, and that there is no room in his formulation for the labor theory of value.

Another level of criticism involves an apparent lack of comprehension on the part of some critics of the logic of the transformation curve. In one memorable article, A. Boiarskii argues that, because an automobile assembly plant can manufacture no cotton textiles and a textile plant can manufacture no automobiles, no rate of transformation exists between the two products.[2] Obviously, says Boiarskii, there is a "qualitative difference" between textiles and automobiles which Kantorovich ignores.

This criticism is a manifestation of a general and pervasive suspicion among the orthodox of the attempt to compress the complexities of economics into a few mathematical expressions. Many object to the abstraction which forms a necessary part of mathematical model-building; these critics somehow feel, but never demonstrate, that important variables are slighted in the process of abstraction. Here, again, ill-defined "qualitative aspects" of economic reality are invoked to demonstrate the inadequacy of mathematical categories.

In the same vein is found opposition to alleged "oversimplification of criteria of optimality" in mathematical analysis. P. S. Mstislavskii argues that the criteria of minimal costs and maximum output neither reflect the full needs of socialism nor satisfy the laws of socialist economics.[3] A. I. Notkin's

---

1. Phrase from Alec Nove, The Soviet Economy (New York: Praeger, 1961).
2. See Alec Nove, op. cit.
3. V. S. Nemchinov, ed., op. cit., p. 257.

viewpoint is the same: minimum cost and maximum output as criteria for economic optimality is "a non-historical, and too abstract a formulation."[1] In suggesting additional relevant criteria, Notkin adds optimization of the proportion between accumulation (investment) and consumption by the population. This particular criticism will be examined in detail in the discussion of objections to the static nature of linear programming.

There is another group which falls within the limits of orthodox Soviet thought, yet which is less hostile to the application of mathematical methods in economics. Economists in this group find the allocative logic of linear programming compelling and see mathematical methods as a fruitful line of investigation for economists. They stop short, however, of a recognition of the full significance of the analysis since they demand that mathematical analysis be limited to local application, e.g., in workshops, factories, and regions. Academician V. S. Nemchinov, a central figure in the discussion of the applicability of mathematics in economics and a vigorous defender of the usefulness of mathematical techniques, shares this view.[2]

The advantage of this position under Soviet conditions is that, under limited application of the system, the o. c. e.'s associated with the rates of transformation evolved in linear programming analysis cannot be as easily construed as general equivalents of exchange among goods as they might be under a broader or more general application. This advantage is particularly helpful to the Soviet economist who is anxious lest he lose sight of the real source of value to the detriment of his reputation and self-esteem.

The costs, however, to national-economic efficiency of partial as opposed to general application of the system are high. Thus, as has been noted earlier, both Novozhilov and Kantorovich insist again and again on the necessity of a general application of their systems in order to reap the full benefits of mathematical analysis.

Thus far, only criticisms ultimately based on doctrinal objections to the marginal solutions involved in linear programming have been reviewed. Another class of objections, the second noted by Novozhilov, will now be considered. These criticisms question the suitability of linear programming—a static form of analysis—as a technique for the solution of problems involving long-run or dynamic planning. A great deal of Soviet criticism on this score seems to stem from a basic misunderstanding of what it is that linear programming purports to do.

A. Kats attacks Kantorovich's model, arguing that "major miscalculations" are likely to be made if resources are allocated over a long period

---

1. Ibid., p. 191.

2. See V. S. Nemchinov in Foreword to L. V. Kantorovich, op. cit., p. 7.

according to o.c.e.'s.[1] He alleges that "technological repression," i.e., systematic biases against the growth of technology, is bound to result from period-by-period "optimal allocation." In Kats' opinion, if labor is continually allocated according to the demands of present-period optimality its productivity growth over time will be less than if it were non-optimally allocated. This situation occurs because optimal allocation will involve the employment of some labor in conjunction with relatively "backward" capital equipment. The labor thus employed suffers by the experience, apparently in some educational sense, and labor productivity rises are forestalled.[2]

This may be quite true. It does not, however, constitute a demonstration of the inapplicability of static linear programming techniques in a dynamic situation. The possibility of dynamic non-optimality resulting from the use of o.c.e.'s is explained by the fact that no relationship between kind of capital employed and labor productivity growth is included in the system which Kats takes to be in use. Indeed, Kats could substantiate his criticism by demonstrating that the gains in labor productivity resulting from (mis)allocation of resources into capital goods of a technologically advanced or more educational sort more than offset present-period losses chargeable to misallocation. In doing so, however, he would in effect be positing a relationship between labor productivity and labor's association with certain kinds of capital equipment. This relationship could be internalized in the analysis, and a new dynamic scheme of optimum allocation could be determined. In other words, the unmanageable exogenous variable could be made endogenous and an integral part of the analysis.

The great problems attending dynamic programming as a result of changes or the possibility of changes in exogenous variables should not be underestimated. Growth in the labor force, discoveries of new supplies of resources, and new technologies are all changes in the parameters which must be considered in the dynamic analysis, and which may not be adequately anticipated. The more frequent are the revisions of the data employed in generating economic optima, the more likely it is that such changes will be reflected in the parameters. It is clear that this "constant revision" approach yields less satisfactory results than might have been attained if all changes in exogenous parameters over the plan period had been predicted. But criticism of linear programming's failure to deal with truly stochastic elements must be judged irrelevant, because no system deals effectively with them.

A second criticism of dynamic programming plunges deep into the rationale of a planned economy. In another article, Kats asserts that before

---

1. See A. Kats in V. S. Nemchinov, ed., op. cit., and "Concerning a Fallacious Concept of Economic Calculations," Voprosy ekonomiki, No. 5 (1960). Translated in Problems of Economics, Vol. III, No. 7.

2. As a Marxist, Kats limits his understanding of technological growth to growth in labor productivity.

o. c. e.'s are generated, and before the extremal problem of yielding minima and maxima is solved, the optimal relationship between consumption and investment must be determined; in other words, the choice between present and future consumption must be made. When Kats argues that Novozhilov and Kantorovich "leave this most fundamental problem behind the door"[1] in their calculations, he could not be more correct. Novozhilov relies in his analysis, it will be remembered, on a stipulated final bill of goods; Kantorovich promises to maximize output of a given assortment of goods. Both agree to maximize national-economic ophelimity after being given a set of rules. Neither makes these rules, and society in general is not allowed the luxury of a subjective, anarchic process of rules formation. Yet a given path of development or rate of growth can be shown to be optimal, or preferable to all others, only if preferences are elaborated. Preferences can be "objective" only in relation to still other (exogenous) constraints. The search for the fully objective solution to this basic problem of dynamic planning must be ultimately fruitless; for, in the "efficient" solution, that which is society's future must be defined, and income in this future must somehow be weighted relatively to present income. Neither Marxism nor the tools of economic analysis can find the solution to these problems.

Aside from theoretical and doctrinal objections, there are important and relevant difficulties in the practical application of linear programming on a large scale. More specifically, it would seem that certain properties of the real world militate against the large-scale application of mathematical analysis.

The sheer size and complexity of the Soviet economy are crucially relevant in this connection. The vast numbers of processes, as well as intermediate and final products, demand that the plan include large numbers of mathematical expressions. Despite Soviet optimism about the potential capacity of electronic computers to deal with such systems, the quantity of data still must be reduced to keep it within manageable bounds. The aggregation of data is therefore a necessary part of the planning process. When aggregates are composed of items which are not perfectly homogeneous, plans which employ these aggregates are less precise and therefore less efficient than plans involving a lesser degree of aggregation. At present, the aggregates used in Soviet planning are no more than rough sketches of the real output which they represent. The coefficients relating these aggregates are therefore little more than arithmetic conveniences, bearing little relation to actual processes of production.

It is therefore evident that the Soviets are extremely far from possessing the data collection and processing apparatus which would be necessary to institute even a very crude system of o. c. e.'s.

---

1. A. Kats, "Ekonomicheskaia teoria i primenenie matematiki v ekonomike," Voprosy ekonomiki, No. 11 (1960), p. 92.

The costs of constructing and running such a system would undoubtedly be great, possibly great enough to offset the gains promised by its utilization. With this fact in mind, it is not difficult to understand Soviet opposition, based on purely practical considerations, to the application of mathematics on an economy-wide scale. In addition, the basic assumption of linear programming, that is, linearity of production processes, may do violence to reality in practical application. From empirical studies of a number of industrial processes, it is known that the linearity assumption is valid in a large number of cases over a relatively wide range of output for each process.[1] Returns to scale of plant, however, provide an example of the phenomena of which a statie linear model cannot take account. In this connection, it is interesting to note that Kantorovich has already begun thinking in terms of non-linear models. In a footnote to his presentation in "Obshchie voprosy,"[2] he notes the significance of the non-linear production function.

In the following section, the prospects for mathematical economic analysis in the USSR and some of the implications of the current debate are discussed.

5. Conclusion

Throughout the literature of economic development, one may find arguments to the effect that, if growth is held as a major goal of the system and the system is at a fairly low level of development, significant deviations from "rational" economic behavior may be justified or even demanded. Nove[3] argues that the most efficacious development strategy may well involve a rethinking of the traditional concepts of efficiency, that truly optimal allocation of resources, given certain goals, may conflict with the traditional marginal solution. This may be so, but again, it is true only to the extent that the parameters determining the marginal optima fail to include relevant relationships within the system. Montias has pointed out the complementarities involved in the early stages of economic growth.[4] Rosenstein-Rodan has called attention to the external economies generated by alternative patterns of development.[5] All these concepts, however, can be reduced, from a planning

---

1. Robert Dorfman, op. cit.

2. V. S. Nemchinov, ed., op. cit.

3. Alec Nove, op. cit., Chapter 12.

4. J. M. Montias, in lecture, Spring 1963.

5. P. N. Rosenstein-Rodan, "Problems of Industrialization of Eastern and South-Eastern Europe," The Economic Journal (June–Sept.), 1943. Reprint., The Economics of Underdevelopment (Agarwala and Singh: London, 1958).

standpoint, to poorly predicted, unforeseen, or unforeseeable relationships in terms of a mathematical model. It is true that a scarcity price system is unlikely to take such relationships into account; and a planning board which uses linear programming as an allocative tool, a "market in reverse," will probably also fail to consider them.

Thus, if the Soviet Union is still in this "development stage,"[1] possibly, it may profit by disregarding the marginal calculus. But the "development stage" bears within it the seeds of its own destruction. Soviet economics, therefore, cannot ignore forever the logic of scarcity. Gradual recognition of the usefulness of the margin in resource allocation will bring about greater acceptance of the methods which employ it. At a recent conference on the application of mathematics in economics, Nemchinov for the first time noted that there are economic problems which require Kantorovich's objectively conditioned estimates for their solution.[2] At the conference, the doctrinal opponents to mathematics were conspicuous in their absence; an overwhelming majority of the papers delivered were by mathematicians and technicians versed in the operation of electronic computers. A tentative conclusion therefore suggests itself. The traditional categories of Soviet economic thought will not be revoked; economics will remain the province of mystics and philosophers while the work of economics—planning and production—is carried on by technicians who know only their job, and who are not concerned with the nature and source of value.

---

1. The term is Nove's.

2. "Konferentsia po primeneniu matematicheskikh metodov i EVM v planirovanii," Voprosy ekonomiki, No. 3 (1963).